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ACI
PROCEEDINGS
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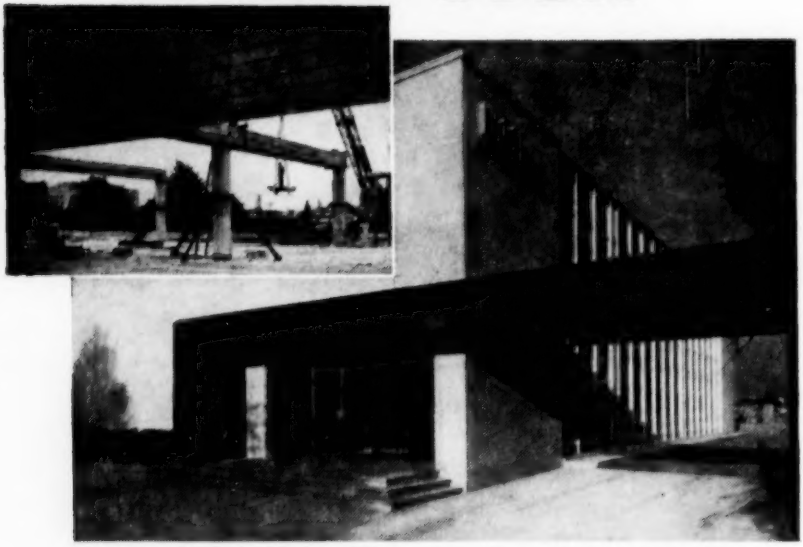

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JANUARY 1957

No. 7

Vol. 28

PROCEEDINGS VOL. 53

JOURNAL
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Contributions to the Journal

The ACI JOURNAL technical pages are a medium for reporting what is of current interest in the concrete field; in research, design, construction, maintenance, or manufacturing. Members and readers are urged to "have their say" as a means of sharing knowledge and experience; time and changing interests demand that new names and new ideas take their places beside the old. Contributions describing new materials and techniques or new applications of familiar practices are welcomed by the Institute.

To guide the flow of material the Board of Direction has assigned to the Technical Activities Committee the selection of papers, committee reports, discussions, and other contributions for JOURNAL publication. To assist in their technical appraisal a corps of experts in various segments of the field volunteer their services. From the technical standpoint, acceptable contributions present original material for improvement of design, manufacturing, or construction; confirm, revise, or upset established ideas or practices; or review, digest, or arrange accumulated experience for more ready use.

General considerations by TAC are whether the contribution will be of interest and service, will fit into a well-balanced publications schedule, and can be published at reasonable expense to the Institute.

To expedite appraisal and publication, if accepted, contributors are requested to furnish glossy prints or inked tracings of illustrations accompanying manuscripts submitted for consideration. Further details of form and physical arrangement of contributions begin on p. 14, 1955 *ACI Directory*. Separate prints of the "publications policy" are available free upon request to the Secretary.

Manuscripts of papers, discussions, and reports should be sent in triplicate to:

Secretary, Technical Activities Committee
AMERICAN CONCRETE INSTITUTE
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In ACI Journal next month

"Proposed Recommended Practice for Design of Concrete Pavements" reported by ACI COMMITTEE 325, leads offerings in the February JOURNAL. Combined and condensed from several subcommittee reports printed separately in the summer and autumn issues of the JOURNAL, this final version offers comprehensive directions for design of rigid airport and highway pavements or bases. Design practices of all principal state and federal agencies concerned with paving have been reviewed and correlated within the limits recommended by the committee.

JAMES S. MINGES and DONALD S. WILD are co-authors of "Six Stories of Prestressed Slabs Erected by the Lift-Slab Method." Structural design and construction procedures for the Litchfield County Hospital at Winsted, Conn., are treated, with emphasis on prestressed slab design. Special lift-collar design enabled welding of collars to steel columns to be done after all lifting operations were completed.

Five remaining papers to appear in February are from a symposium on shrinkage sponsored by ACI Committee 209 at the Institute convention in Philadelphia last year. First of these, by R. E. COPELAND, is "Shrinkage and Temperature Stresses in Concrete Masonry." He presents a theoretical method of locating control joint spacing in blank masonry walls.

Second paper from the symposium is "Some Factors Influencing Shrinkage of Concrete Pavements," by F. N. HVEEM and BAILEY TREMPER. The writers have focused on the role of clay in aggregates as a factor influencing shrinkage of concrete. California Division of Highways field tests for evaluating activity and quantity of clay contained in aggregates are cited as useful means of specifying limits which facilitate control of drying shrinkage.

A 16-year "Study of Shrinkage in Concrete Frames" is reported by MORGAN B. KLOCK and ROBERT R. SHERIDAN. Frames, 275 ft long, were constructed without contraction joints. Carlson electric strain meters buried in the concrete in several locations, have indicated drying shrinkage of some 600 millionths horizontally and 1100 millionths vertically.

WILLIAM LERCH, author of "Plastic Shrinkage," asserts that plastic shrinkage and plastic shrinkage cracking are caused by a too-rapid evaporation of water from the surface of fresh concrete. Proper construction procedures can readily minimize this type of shrinkage cracking.

"Determination of Creep Strain of Concrete under Sustained Stress," by F. EUGENE SEAMAN, presents the many problems involved in measurement of creep strain in concrete as distinguished from elastic strain and other volume changes. Limited data indicate curing methods may greatly influence the creep strain property of concrete.

Title No. 53-33

Proposed

Manual of Standard Practice for Detailing Reinforced Concrete Structures*

Reported by ACI Committee 315

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SYNOPSIS

The manual presents recommended methods and standards for preparing drawings for the fabrication and placing of reinforcing steel in reinforced concrete structures. The previous ACI Standards ACI 315-51 (detailing of building structures) and ACI 315A-53 (detailing of highway structures) have been combined into one manual.

Typical engineering drawings (and, for buildings, placing drawings as well) illustrate the use of the standards.

The American Concrete Institute previously published separate manuals for the detailing of reinforced concrete building structures [*Manual of Standard Practice for Detailing Reinforced Concrete Structures (ACI 315-51)*] and reinforced concrete highway structures [*Manual of Standard Practice for Detailing Reinforced Concrete Highway Structures (ACI 315A-53)*] because of slight differences in procedure in these two fields. Because there were so many items common to both the buildings manual and the manual for highway structures, and a demand to have both manuals available for ready reference,

*Title No. 53-33 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 28, No. 7, Jan. 1957, *Proceedings* V. 53. Separate prints are available at 35 cents each. **Discussion** (copies in **triplicate**) should reach the Institute not later than May 1, 1957. Address 18263 W. McNichols Rd., Detroit 19, Mich.

This report was submitted to letter ballot of the committee which consists of 22 members; 22 members returned their ballots, of whom 22 have voted affirmatively and none negatively. It is released by the Standards Committee for publication and discussion with view to its consideration for adoption as an Institute Standard to supersede the "Manual of Standard Practice for Detailing Reinforced Concrete Structures (ACI 315-51)" and the "Manual of Standard Practice for Detailing Reinforced Concrete Highway Structures (ACI 315A-53)" at the 53rd Annual Convention, Dallas, Tex., Feb. 25-28, 1957.

Committee 315 decided to condense them into a single manual. The main changes in the new manual from the two previous ACI standards are summarized herein.

ACI 315-51 and ACI 315A-53 were combined into one manual. In general this was done by combining the descriptive material in the early part of each manual into a single write-up, eliminating any duplication between the two manuals and making separate descriptive articles under each main heading when there was sufficient divergence between building practice and highway practice to make this desirable.

The two main branches were differentiated for reference purposes as *buildings* and *bridges*, grouping under "buildings" all structures which would come under the city building code and under "bridges" all structures which would come under a state highway department.

Tables, text, and drawings have been corrected to embody the revisions of the new ACI Building Code (ACI 318-56). In this connection, since this is a detailing manual and not a design handbook, it was not felt necessary to change the number and spacing of stirrups or the cutting and bending of bars. The engineering drawings and shop drawings are in agreement, but in some cases it is doubtful whether an eager designer could check back and determine whether the positive and negative reinforcing bars, their extension, and the number and spacing of stirrups are all consistent. Such corrections would have required tremendous effort and would not improve methods of delineation.

One of the drawings added is an alternative method of delineating a concrete design, substituting elevations of the runs of beams for the schedule form. The drawing showing the details of beams in schedule form is, of course, also retained. Many designers feel that the alternative method has advantages in (1) reducing the total time of making such drawings, (2) being to some extent self-checking because of the easy comparison of positive and negative resisting moments, (3) providing a mental picture of a moment curve that determines bending and cut-off points for bars, (4) simplifying the preparation and checking of shop drawings, and (5) making for easier verification of placement in the field. The schedule form remains the best way of preparing the shop drawings.

The other major change is the addition of a chapter which considers the designer's viewpoint. The chapter was added because it was found that in addition to use by bar company detailers, the manuals were proving of considerable value to architects and consulting engineers as to standard practices in reinforcing steel and best methods of conveying design information to detailers. Since this is new text, and not merely an editorial revision of text in the earlier manuals, this chapter is included herein.

CHAPTER 5—DESIGN DRAWINGS FOR BUILDINGS

Although this manual was prepared mainly as a guide to reinforcing bar detailers, it has proved useful to designers in architectural and engineering

offices. Therefore, this chapter on desirable methods for describing design requirements has been added.

Concrete being weak in tension, reinforcement is tailored quite accurately to take care of all tensions envisioned by the designer whether direct flexural (main reinforcing bars) or diagonal (stirrup reinforcement). The designer has full data on the assumptions he made. Only he knows just how and where reinforcement is needed; only he can supply such information to the detailer. He may be able to do this for the case of simple, fairly regular structures by schedules, rules, ratios, and diagrams. However, more and more structures are being analyzed, at least approximately, as rigid frames. Unless live loads are low in proportion to dead loads, unless spans, end conditions, and restraints are reasonably uniform, arbitrary rules about bending of bars may prove decidedly *unsafe*. Actual structures may not behave exactly in accordance with the design assumptions, which necessarily idealize them a bit, so allowances must be made for possible variations. The most carefully prepared design can be spoiled by failure to reinforce a critical point properly. There is a great deal more to making a satisfactory design than the mere computing of maximum positive and negative flexural reinforcement. The real skill comes in developing satisfactory details.

Design drawings must be complete to the extent that every bit of information regarding the size and arrangement of concrete members, and the size, positioning, and detailing of reinforcing bars is completely covered, either by a drawing, description, diagram, note, rule, or reference to a standard manual. It is not proper for the designer to exercise individual preferences in checking shop drawings, or to rely upon ambiguities or the knowledge and skill of contractors and subcontractors to take care of items not properly covered in the contract documents.

It is not necessary for the designer to make complete drawings of each member nor to schedule completely the bending of bars in each member, but schedules should be so prepared that this information is determined unequivocally by diagrams, notes, and references.

The sample drawings in this manual are shown as a method of presenting information, not to establish standards for design matters such as the bending points of bars. The drawings are intended to illustrate that it is the designer's function to tell the detailer specifically what he wants and needs. The designer has the computations, moment diagrams, and the whole basic philosophy immediately in front of him, and it is his responsibility to define the requirements for the detailer.

While with uniform loads and equal spans, it may be satisfactory to bend bars at the seventh-point and fifth-point and to extend them to the quarter-point (as shown on Drg. 18*), this is not a sufficiently safe general practice to consider it standard in the absence of any information. A notation that shop detailing is to follow the standards in this manual should not be extended

*Because of page-size limitations the drawings referred to in this chapter are not reproduced in this abstract. —Editor.

to apply to such matters. Note also that ACI 318-56, Section 902(a) requires negative reinforcement to be carried $L/16$, d, or half-bond length past the extreme position of the point of inflection.

The designer should include in every set of plans a typical slab, joist, beam, and column detail, showing the exact arrangement of reinforcing steel desired, such as is illustrated in Drg. 14, with the statement that unless otherwise provided for in the schedule, this arrangement of reinforcing steel is to apply. Every dimension on this drawing is important and serves its purpose.

Graphical representations

Graphical representations are to a considerable extent self-checking. Many things are immediately apparent on a scale diagram that are hard to sense from a column of figures. So with continuous beams and quadrangular frames, the termination, bending, lapping, or extending of bars are more easily checked on a scale elevation than in a schedule, especially where the scheduled members involved are not in sequence. A scale elevation of a whole line of continuous beams shows immediately, in a manner that is almost impossible to grasp from a standard schedule, that the positive and negative moment requirements have been properly met at each point.

Whenever a designer finds it necessary to make sketches for his own use to determine some of the data or whenever complicated notes have to be worked into the schedule to convey this information, it will be found safer and quicker for designer, checker, and detailer alike, to prepare true-scale elevations of the runs of members affected and show precisely what is wanted. Such layouts should not be too schematic but reasonably true. The designer and his checker can then spot at a glance inconsistencies that might never be caught otherwise. Drg. 19-A illustrates this method applied to the building scheduled on Drg. 18.

Very often, especially in light-occupancy buildings such as schoolhouses, hospitals, hotels, and apartments, there is the case where a wide room either side of a narrow corridor, makes it necessary to provide for negative moment across the short center span. This may be done in several ways, some of which are illustrated in Drg. 15. The detailer should not be required to determine the extension and cutting of bars. There are several ways for the designer to convey to the detailer the needed information on bar bending. One is a note in the schedule that the truss bar of the outer span is to extend through the top of the corridor to a certain point of the opposite span. If this provides excessive steel in the middle of the corridor, the note could provide for the truss bar to extend half a lap distance past the centerline of the corridor to splice with the opposite bar. If this method is used, then the corridor span would be scheduled only with straight bottom bars, the top bars having already been covered.

A second manner of covering this same situation is on a schematic diagram (Drg. 15). It is not necessary for the designer to work out all the

dimensions (though it is certainly his privilege if he cares to do so), but it is definitely his function to show at least in terms of percentage of span where he wishes bars bent and to what point extended. He can do this readily while making the design, and, incidentally, it will save his own time in checking shop drawings, especially if it should be necessary to resubmit them repeatedly.

Schedules

A schedule is a great time saver and great space saver when used for standard structures with relatively low live loads, reasonably equal spans, and uniform column sizes and spacings, and is particularly effective on placing plans when good engineering drawings have already been prepared. Such a schedule needs to be supplemented with diagrams establishing the minimum fireproofings, termination and bending of bars, and similar data. Heavy live loads, widely differing adjoining spans, variations in column sizes and spacings, and fluctuations in relative stiffnesses can move the zone of contraflexure considerably, thereby affecting the termination, extending, and bending of bars greatly. Such lack of regularity in framing could necessitate carrying top bars the full length of a short span located between two long ones, or may make it impracticable to bend any bars at all, or may require stirrups or ties the full length of a member. Conversely they may make it possible to bend up considerably more than half the bottom bars in a heavy girder, anchoring and terminating them in a compression zone.

More good design skill is required in planning all phases of the reinforcement throughout the length of a member than in selecting top and bottom bars for maximum moments. This is the province of a designer, and a good one at that, certainly not of a detailer. A design only partially delineated is only partially complete. Relying on codes, manuals, or standard practices to bridge the gap is careless, unfair, and inefficient.

A designer is entitled to use any arrangement of reinforcement he sees fit, *provided* that he makes his wants clearly and completely known on the design drawings and that his design is practical for use in the field. Failure of the designer to set forth any special requirements clearly makes him subject to accepting, within reason, what the detailer offers as his interpretation.

The designer should avoid possible ambiguities. Scheduling top bars should be done by reference to the number of the columns over which they occur, as on Drg. 22, not by scheduling them in one or another of the continuous beams. It is almost impossible to write a clearcut note that will describe bars projecting from the top of one beam into the top of its neighbor, especially when the meeting beams may not always be the same pair. Either call for extra top bars on the plan at the positions they occupy or in the schedule by the columns over which they occur.

Since negative moments drop off rapidly and since top bars are frequently used only for reinforcement in addition to the truss bars, it is obviously not

necessary that all of this steel run from the quarter-point (or whatever it may be) of one span to the quarter-point of the other. The designer should state his needs. Since different factors determine different lengths, it may be necessary to establish two or three limitations, such for example, as extending to the eighth-point of the span, but not less than, say, 24 diameters past the face of the support. If there are several bars, it might be that one should go to the quarter-point and another to the eighth-point.

The designer should state whether his steel is in one or multiple layers. If all the steel is in one layer, nothing need be said, but if the steel is designed in two or three layers, that fact should be so noted.

There should be a clear statement of the amount of anchorage of bars. Bottom bars, for example, may extend 6 in. into the support as on Drg. 14; wall ends of truss bars, 17, 21, or 24 bar diameters into the support, either straight if possible, bent if necessary. If bars are of lengths that would be awkward for shipment, the designer would do well to indicate in what zone splicing is permitted or in what zones it is not desired. Diagrams and notes should clearly indicate whether reference is being made to the centerline of the support or to the face of the support, whether references are to the clear span or to the center-to-center span.

All of these items need be stated only once. They can be shown on a typical diagram, and then designer, detailer, and the field know what is required.

Schedules should be prepared equally carefully and no abbreviations or symbols should be used without explanation. Frequently, "B" is used for the width of a beam stem, occasionally for the width across the tee. Sometimes a beam depth is measured under the slab, more often through it. Sometimes it is the effective depth, but more often the total. A simple diagram, Drg. 14, will make clear to everyone exactly what is meant by these symbols.

Stirrup spacings are usually recorded from the face of the support (occasionally from the center of the support), but a diagram would make this clear. It has been known that stirrups have been incorrectly spaced both ways from the center of the span by an inexperienced erector. Note that ACI 318-56, Section 801(d) requires two additional stirrups at $d/2$ beyond the point where stirrups are not needed by computations and in freestanding rigid frames is much more severe in the matter of extending negative bars and providing web reinforcement.

Simple sections of multi-rodded columns, showing the preferred arrangement, take but a moment to prepare and are much more satisfactory than arguing later as to what is standard. Detailers should not be perfecting the design, but merely carrying out the designer's intentions.

A few notes will clear up a great many points. There should always be a notation as to the quality of concrete and the grade of reinforcing bars upon which the design is based, the live loads which the structure is capable of carrying, as well as the assumed capacity of foundation soil, and notes about any loose lintels or other miscellaneous items not readily apparent. The

experienced designer obtains more favorable bids by stating clearly who is to furnish unusual items such as threaded dowels into structural steel members which belong in the structural steel contract; or reinforcing bars which are used as bridging for open web steel joists, which belong to the joist vendor.

The designer will often do well to indicate at least some of the more obvious construction joints so that all of the trades are working along the same line. In a box culvert, the invert is usually cast as the first operation, the walls later, and the top slab often still later. The experienced designer can indicate shouldered joints reinforced to take moment, shear, and thrust and show the lapping and splicing of bars clearly and without ambiguity rather than simply schematically suggesting the amount of steel required at a few points.

Unusual structures, such as rigid frames, are best handled by making detailed drawings rather than by a schedule. A schedule is helpful only when all parties are thoroughly conversant with the details of what is being described and when all of the items in the schedule are reasonably similar in character.

So far as possible, any one part of a structure should be completed on one drawing. It is much easier for all parties if a floor plan and its accompanying sections and schedules are all on one sheet. The skillful designer can make his needs clear in a way which is easy for estimators, detailers, and builders to follow without undue cross-referencing, and thereby obtain a more economical and better job.

Design notes

The designer should be careful about ambiguous statements. One common fault is a so-called typical section which is not typical of anything. Any section should be an absolutely true section at a designated point and then as many other similar sections can be related to it as can be clearly covered by notations. If the related sections differ considerably they should be redrawn and not covered by an excessive number of notations on a "typical section." Specific notations are much better than general ones. There will be less errors and a better job if a condition is reported as applying to columns 1, 2, 7, 11, and 15 only than if a rather vague note is added "applies to all columns unless otherwise noted or obviously unintended." The habit of clear, precise thinking and delineation grows upon one and orders become concise, clear, complete, and unambiguous.

Be on the lookout for possible misinterpretations. A note "16 bars 2 ways 2 faces" is totally unintelligible. This could mean 16 bars each way each face, total 64, or 4 bars each way each face, total 16, or almost any other combination in between. It is just as easy to be explicit.

It is assumed that no reputable designer intends to misrepresent his needs in hopes of obtaining a low figure and then demanding additional material, but that the ambiguities arise solely through the lack of realization of how easily a carelessly written notation can honestly be interpreted differently from the writer's intention.

Leave no incomplete statements. Make them clear, complete, concise. Descriptions are best given in the imperative mood: "do this," "supply this," "bend these bars," rather than "these bars may be bent" or "this may be done." There is a different attitude on the part of the writer by assuming definite responsibilities for his statements and on the part of the reader in feeling that he has definite instructions.

Use symbols and abbreviations that are standard in standard technical textbooks, and even so explain wherever there is doubt. Improper abbreviations or symbols can often be misinterpreted in ways that sometimes prove embarrassing to the designer.

For pits, holes, depressions, and other odd framings indicate approximately how the designer wishes these treated. This can be done schematically or in accurate detail, but should not be left for the detailer to guess at.

In all but the simplest structures, separate framing plans are desirable. It is practically impossible to superimpose the structural elements upon the architectural plans without confusing both. Time will be saved for the designer, better jobs will result, costs will be reduced, and time will be saved in checking shop drawings, if the structural elements are shown separately. Structural framing plans should have sufficient dimensions for laying out the structure and for estimating and detailing. The size, shape, arrangement, and location of every structural element should be shown clearly and completely. Sufficient sections should be drawn to establish clearances around all of the architectural treatment, ducts, mechanical equipment, and other interferences.

Caution—Concrete being notably weak in tension, designers are cautioned to check carefully all points where tension values (direct or diagonal) may run high, e.g., in footings around the base of columns, in flat plate floors around the supports, in rigid frames and continuous beams near the point of inflection, in beams loaded heavily mostly in one side where torsion develops, in cantilevers, reentrant corners, and the like. Reinforcement must cross all planes of probable fracture to be effective, and be securely anchored on either side, and quite a few designers are inclined in cases of high tension to put the entire stress on the steel rather than the excess over v_c.

Discussion of this paper should reach ACI headquarters in triplicate by May 1, 1957, for publication in the Part 2, December 1957 JOURNAL.

Title No. 53-34

Much can be learned from structural failures. The ACI JOURNAL here presents a three-part symposium on the recent AMC warehouse failures. Part one discusses design considerations.

Rigid Frame Failures*

By BOYD G. ANDERSON†

SYNOPSIS

The fundamental behavior of concrete members in transferring shear load is not fully understood, and in some cases designs have resulted in costly failures. One such failure occurred when portions of some U. S. Air Force rigid frame warehouses collapsed. General nature of distress is described; materials and construction procedure are analyzed briefly. A critical review of the design suggested some points subject to question. Some general comments are offered on shear design.

All engineers interested in concrete construction are aware of the abundance of excellent research work being conducted in all phases of concrete technology. Engineers are also equally familiar with a variety of experience showing that concrete has been a most reliable and economical engineering material. Nevertheless, in spite of these extensive research programs and in spite of the successful use of concrete in structures of all types, the fact remains that the fundamental behavior of concrete members in transferring shear load, one of the most common design features, is not fully understood, and in certain specific cases design practice has been used that has resulted in costly failures.

The problems involved in determining the shear capacity of reinforced concrete members are complex and are not readily soluble by either theory or testing. To the present time the permissible shearing stresses have been determined by empirical methods derived from laboratory test data obtained from test units which may not entirely simulate either the loading conditions or the structural system occurring in many buildings.

The major differences in design practice in different countries at the present time are that U. S. and Canadian codes assume that the concrete will participate with the web steel in carrying the shear, while most European codes require that the full shear be carried by the web steel when the stress exceeds certain values assumed as representing the shear capacity of the concrete. The German code further requires that wherever the specified shear capacity

*Presented at the ACI 52nd annual convention, Philadelphia, Pa., Feb. 22, 1956. Title No. 53-34 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 28, No. 7, Jan. 1957, *Proceedings* V. 53. Separate prints are available at 50 cents each. **Discussion** (copies in **triplicate**) should reach the Institute not later than May 1, 1957. Address 18263 W. McNichols Rd., Detroit 19, Mich.

†Member American Concrete Institute, Ammann & Whitney, Consulting Engineers, New York, N. Y.

is exceeded the web steel must be carried to the center of the span. Most of the European codes, which assume the use of plain bars, recommend use of nominal stirrups throughout the length of the member and recommend bending up as much of the longitudinal steel as practical.

All present codes presume that the shear strength of concrete will increase either proportionately or in some increment with an increase in the compressive strength of the concrete. The codes also recognize an increase in shear capacity of a reinforced member with an increase in the amount of web reinforcement. However, extensive study of past tests and an increasing

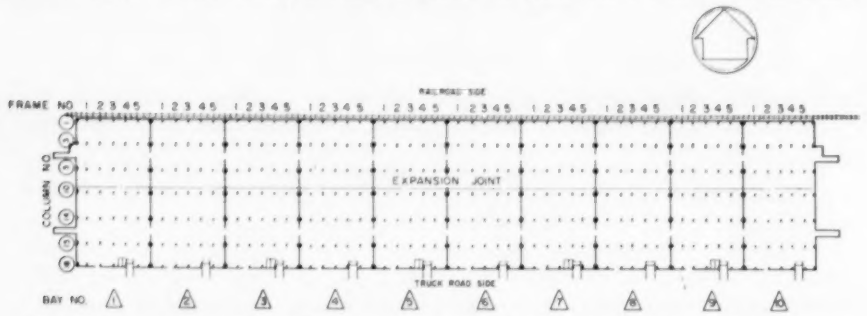

Fig. 1—Plan of special AMC warehouse, Wilkins Air Force Depot, Shelby, Ohio

amount of data from new tests indicate that principal stresses caused by a combination of shear and flexural stress may have an important bearing on the load capacity of the members. Furthermore, individual member tests have not been set up to anticipate the axial forces commonly occurring in actual construction, a factor now demonstrated quite conclusively as having an important effect on the nominal shear capacity of a member. The codes

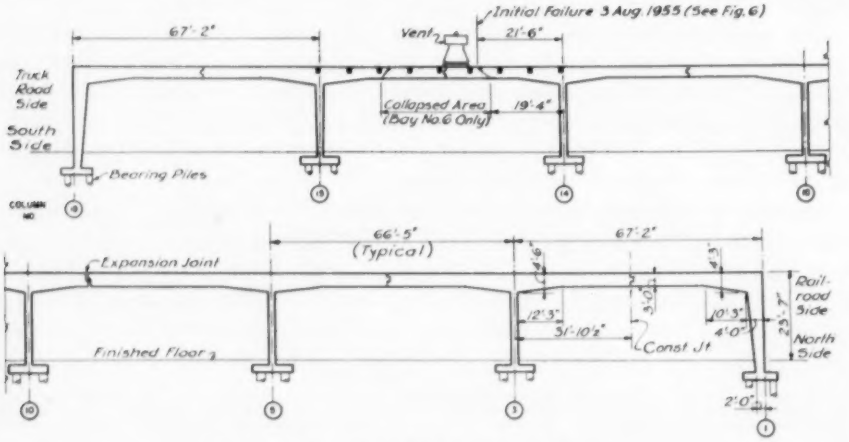

Fig. 2—Elevation of typical interior frame

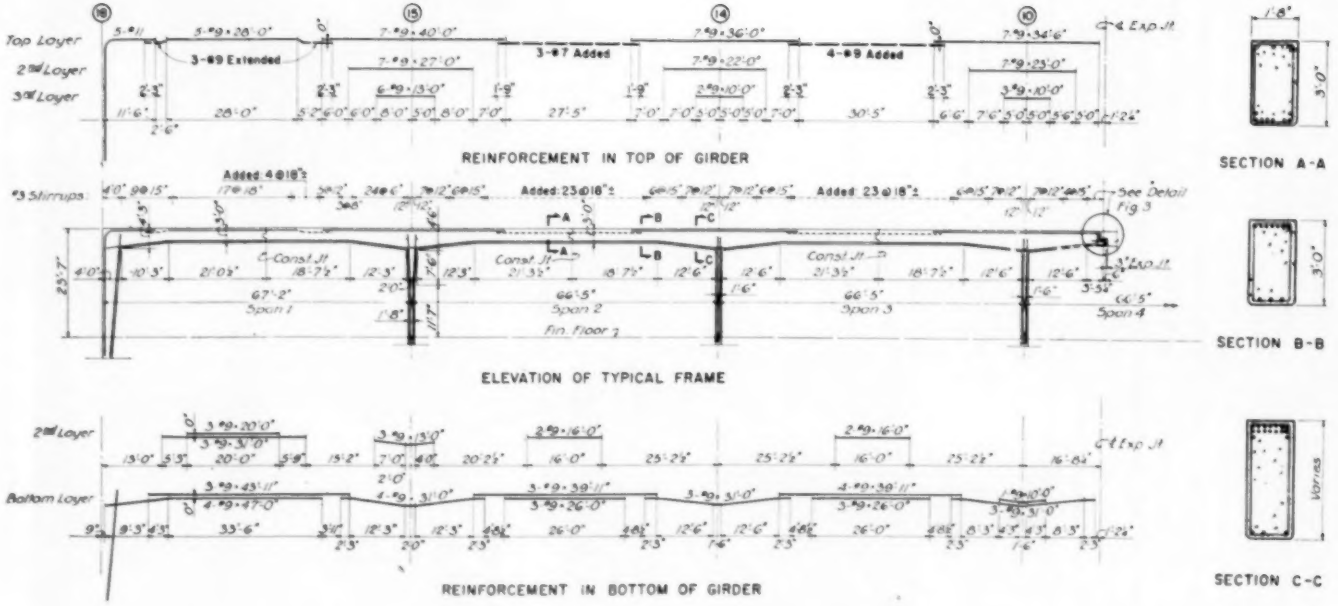

Fig. 3—Typical interior frame, column No. 18 to expansion joint—original design and OCE revisions

also do not consider that the participation of concrete in carrying a portion of the shear load as demonstrated by so many simple span test beams may depend on the presence of flexure acting in combination with the vertical shear force.

With the great number of successful concrete structures in existence and with design procedures spelled out in elaborate detail in the building codes, it is not unexpected that such codes are accepted without question by many

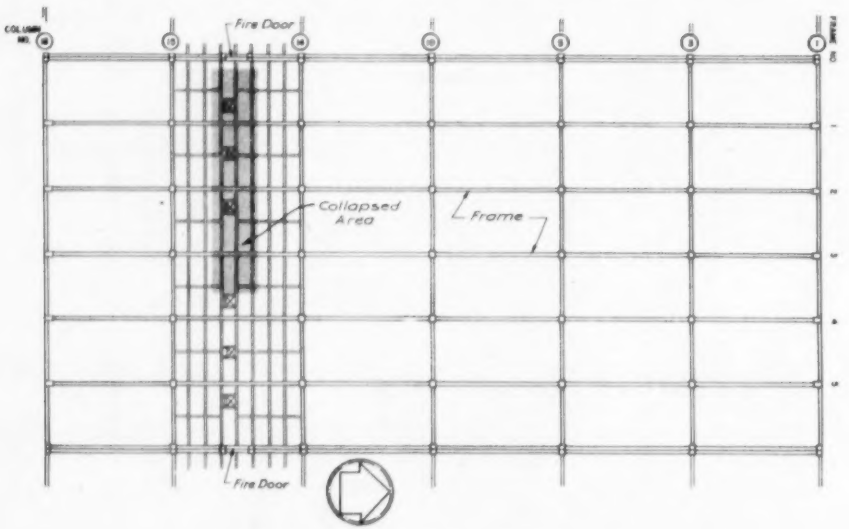

Fig. 4—Plan of bay 6 collapsed area

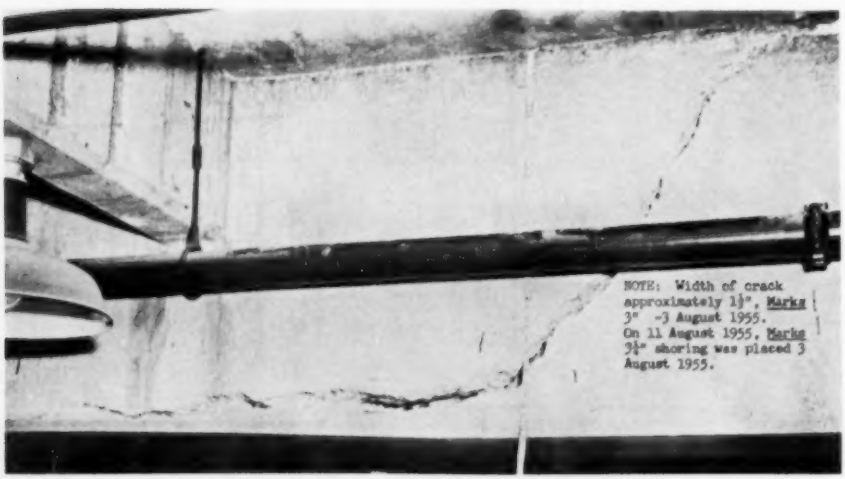

Fig. 5—Cracking in one bent that later collapsed

designers as affording total judgment on the subject. Though strict dependence on the rules has resulted in an overwhelming number of properly functioning structures, it is always necessary to make sure that the structure under design conforms to the conditions forming the basis of the code requirements before these minimum regulations are used as the sole criteria of the design.

Warnings from investigators, as have occurred on the shear design problem, often remain academic and controversial until difficulties are observed in actual construction. One such difficulty recently occurred when three bays collapsed in a special AMC warehouse at Shelby, Ohio. This warehouse was constructed under the supervision of the Office of the Chief of Engineers, Department of the Army, for use by the U. S. Air Force. The following material contains excerpts and certain of the figures drawn from a report on the failures prepared under the direction of OCE.*

Fig. 6—View of failure

The warehouse consists of a six-span rigid frame building 400 ft wide by 2000 ft long (Fig. 1). The building frame and roof covering is separated by an expansion joint near the center of the 400-ft width and by double short span frames spaced at 200-ft intervals along the 2000-ft length. The six spans are formed of haunched rigid frames with each span approximately 67 ft long as shown in Fig. 2. Reinforcement was as shown in Fig. 3. The collapse occurred in three frames of the second interior span as shown in Fig. 4.

Severe cracking had been observed prior to failure in one bent that later collapsed (Fig. 5). This bay was supported at that time on temporary shoring but it later failed when the adjacent bays collapsed resulting in the failure shown in Fig. 6. The failure occurred about 1 ft 6 in. beyond or outside of the cutoff of the negative reinforcement. The original design calculations indicated that a small positive moment existed under all loading conditions at the point of failure.

The remainder of the construction in the warehouse consists of prestressed concrete block purlins spanning between frames and spaced 8 ft 3⅝ in. on centers. The purlins in turn support a 4-in. cast-in-place gypsum slab which embeds the top of the main frames by approximately ¾ in.

*"Structural Investigation of Special AMC Warehouses," final report prepared for Engineering Division, Military Construction, Office of the Chief of Engineers, by Ammann & Whitney, 1956.

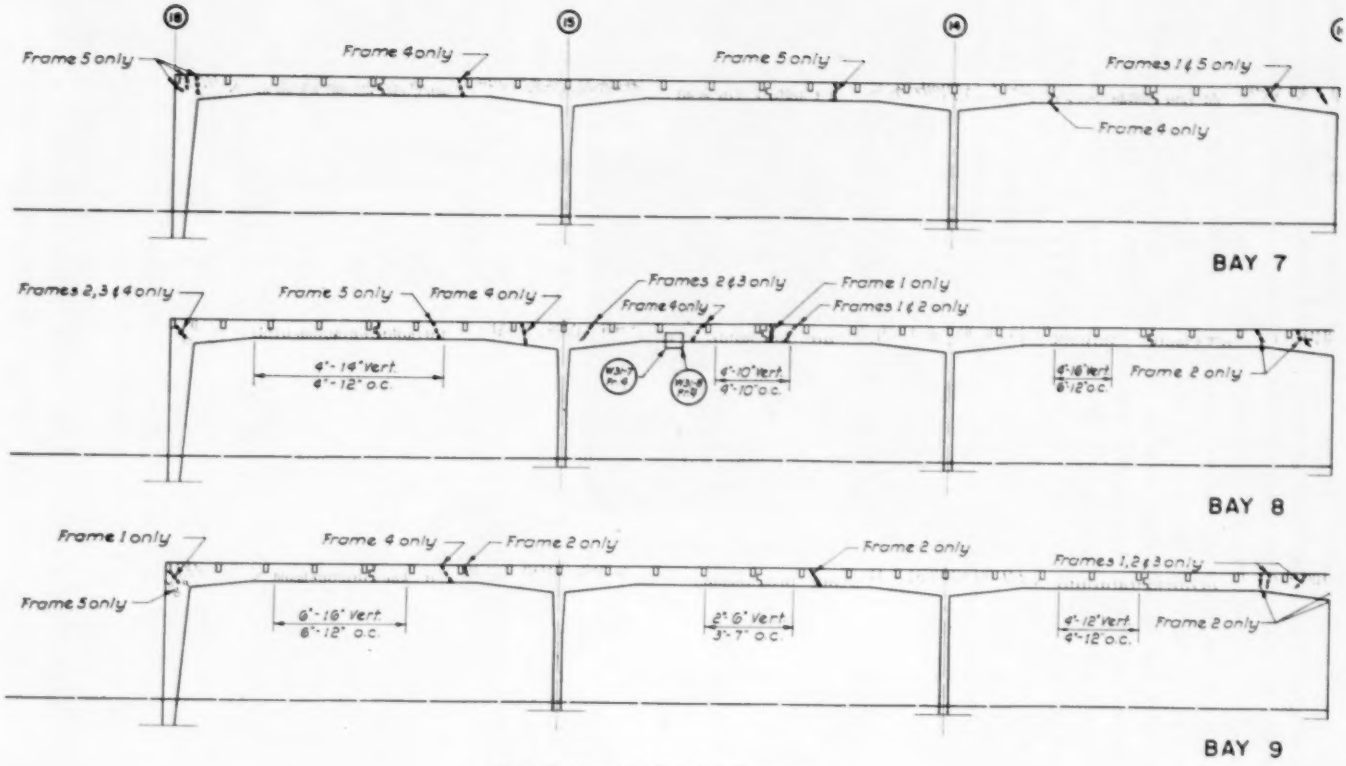

Fig. 7—Typical cracking patterns

GENERAL NATURE OF DISTRESS

The field inspections indicated that the failure was the result of a general weakness resulting in cracking throughout most of the building, though generally the cracking in each bay was most severe in the span that failed. While many of the cracks are hairline in thickness and have little effect on the over-all strength of the frames, the cracks in regions of low moment near the points of contraflexure are of a more serious nature in that no mechanical means is available for carrying the load across the plane of failure. It was therefore believed that all of these particular cracks could progress to a condition similar to that observed in the frames that failed.

Most of the cracking occurred some time after the full dead load was in position, with many of the cracks appearing and increasing in size more than a year after completion of the construction.

The cracks vary in type (Fig. 7) but most make angles of less than 45 deg with the vertical. While most of the cracks appeared to have initiated at the top or bottom of the member depending on flexural conditions, some appear to have started near the centerline.

MODIFICATION TO THE DESIGN

When signs of distress were noted during construction of warehouses having similar framing located at other sites during the construction of this building, revisions were made to the design which included the addition of continuous full length top bars and the addition of nominal stirrups throughout the length of the span design. The structure apparently was somewhat improved by these revisions but cracks did appear in these members. In view of the latest laboratory results, there is still no assurance that these corrections are adequate to provide a desirable factor of safety.

MATERIALS AND CONSTRUCTION

All available data indicate that the quality of the concrete and reinforcing steel used in the construction was in accordance with the specification requirements. Tests show the strength was greater than assumed in the design. Cores for compression tests at the Ohio River Division Laboratories, Mariemont, Ohio, showed a minimum compressive strength of 3455 psi for a specimen taken from the top of the beam near the point of initial failure, with the other cores showing strengths ranging from 4015 to 5240 psi.

Sequence of placing

The concrete of each 400 ft long frame girder was placed in a single placement during one working day. Vertical steel plate construction joints were set at the center of each span prior to concreting. However, the placement was made along the beam and past the plate in a continuous placing operation and the effectiveness of these joints in limiting shrinkage stresses is doubtful. The lengths of continuous placements tended to aggravate shrinkage effects.

Curing

Steam curing was used during cold weather (below 50 F) and curing compound during warm weather (above 50 F). Although the effect of the curing cannot be separated out, it may be noted that the frames concreted with heated aggregate within a heated enclosure in cold weather were later exposed to a sudden temperature drop. The cracking appears somewhat less severe where the concrete was placed during warm weather.

Cement and aggregates

The cement and aggregates used satisfied the applicable federal specifications. The cements were tested by the National Bureau of Standards during construction. The aggregates came from several sources. All the coarse aggregate contained a large proportion of limestone. The fine aggregate varied from crushed limestone to sand containing only 5 percent limestone.

Concrete mix

The concrete mix had the following characteristics: cement factor—517 lb per cu yd (5½ bags per cu yd); average proportions by weight—1:2.24:3.89; and maximum size of coarse aggregate—1 in.

The occurrence of similar signs of distress in other warehouses built according to the same standard plans confirms the behavior experienced in this warehouse. Although some of the materials and methods used in the construction of the various warehouses undoubtedly aggravated the stress conditions in the members, the construction was spaced over too many widely separated contracts to consider that materials and construction methods were the primary source of the trouble.

INVESTIGATION OF ORIGINAL DESIGN

An examination of the original plans and design calculations showed that both the design loads and the design criteria for stress in the members conformed to those ordinarily used for the area and that the concrete and steel detailing satisfied design practice as represented by the minimum requirements of present U. S. building codes. On the basis of the selected design criteria the engineers furnished a thorough analysis based on elastic theory. Specified factors of safety were maintained against vertical loads. Temperature and shrinkage allowances were included in the analysis. It is believed that in civil construction the plans would have been accepted by city building departments.

The critical review of the design suggested the following points most subject to question:

1. The design analysis assumed that the columns were hinged at the base. Although the assumption that the columns were fully hinged at their lower ends resulted in a conservative design for most areas of the members, this assumption was not conservative for all parts of the members. Consideration of partial restraint of the column ends would have improved the distribu-

tion of the steel somewhat and in particular would have accentuated the calculated axial tensile forces developed in the frame.

2. It was assumed by the designers that the expansion joint consisting of one steel bearing plate sliding on another would not develop more than 25 percent of the vertical load as a traction force. Tests have indicated that bearings of this type can develop high values of friction and there is evidence in the structure that high friction forces were developed in this case. Many plates showed no indication of relative displacement since installation, with stress cracks visible in the concrete adjacent to the expansion plates. Furthermore, vertical cracks in the bottom side of a number of frames at the inside face of the heavy exterior frame legs further indicated that longitudinal tension forces may have been developed sufficiently to actually reverse the usual frame moments in the heavy exterior columns. Though some plates moved a portion of the computed displacement, this small displacement may have occurred prior to placing the roof deck and all motion may have been inhibited thereafter.

To check further on this condition, a few of the expansion plates were separated vertically and horizontal motions were observed during the separation. The measured motions during this separation were not of large magnitude, but the lack of motion at this time was not considered too significant as it is possible that the frames had already been relieved by previous cracking and plastic flow and the motion of the frame members was probably restrained by the roof during this test.

It may be pertinent to further note that where earlier cracks had been splinted and filled, the tension forces appeared to have been relieved and the filled joint has consequently remained tight and intact.

3. While the original design analysis followed normal design practice in assuming elastic action of the frames, this assumed behavior can not be expected to predict the precise distribution of internal forces in long haunched concrete members containing variable amounts of compression steel in general or the force distribution in shrinking cracked concrete members substantially restrained at the ends in particular. Approximate analyses made, assuming a shortened cracked section, indicate that redistribution of moment due to volumetric effects could have moved the point of contraflexure substantially toward the center of the span and that substantially higher flexural stresses could be developed in the vicinity of failure than had been assumed. Such behavior would cause flexural stresses as well as axial tension beyond the cutoff point of the top steel.

LABORATORY TESTS

Subsequent to the warehouse failures, a series of laboratory tests was performed on one-third scale models of the warehouse frames by the Portland Cement Association. While these tests are reported in detail in a follow-

ing paper,* it is well to note that the tests conformed qualitatively with both the results of the analytical studies and the behavior observed on the warehouse frames as follows:

(a) Frames having reinforcement similar to the warehouse failed at shearing stresses only slightly above the shearing stresses permitted by the codes for design loads.

(b) When the same frame was provided with added web reinforcement in the region of the point of contraflexure the load capacity approached the flexural capacity of the member.

(c) When an axial tension force was introduced into the frame in addition to the vertical load, a failure similar in appearance and location to that observed in the warehouse was achieved. Failure occurred at a shear stress comparable to the dead load shear stress in the warehouse frame at the time of failure when the axial tension force reached approximately three-quarters of the vertical load on the span.

DISCUSSION

General comments on the warehouse

The nominal shear stress in the warehouse members at the time of failure as computed by usual design methods was well below the shear capacity anticipated by usual design codes.

Comparing the effectiveness of existing United States and foreign codes in preventing failures such as experienced in the warehouse, both the results of the analysis of the warehouse and the results obtained from the PCA test beams make it appear that the German practice of web reinforcement design would have provided adequate shear reinforcement for the building in question. However, it is believed that shifts in the point of inflection due to volumetric effects and inelastic behavior of the members would still require the extension of a portion of the negative steel beyond the limits usually required by elastic analysis. It would appear that design according to other existing codes, American or European, would not require web reinforcement that would have prevented the warehouse failure. These practices would have resulted in dropping the web reinforcement before this point where failure occurred in the warehouse structure. The behavior of the modified warehouse members also shows that the practice of continuing a nominal amount of web reinforcement and longitudinal steel for the full length of the member will not prevent cracking of the member. After cracking such nominal web reinforcement would not be sufficient to carry the shear across the type of failure planes that occur in regions of low flexure and/or high axial tension.

General comments on shear design

The data collected by the ACI-ASCE joint committee on shear, the difficulties experienced in the AMC warehouses, and the PCA test results all suggested the necessity for modification or expansion of the existing code requirements. With this background, immediate steps were proposed to revise the ACI Building Code to help prevent selection of details, design assumptions, and member proportions which would satisfy code requirements

*Elstner, Richard C., and Hognestad, Eivind, "Laboratory Investigation of Rigid Frame Failure," ACI JOURNAL, Jan. 1957, *Proc.* V. 53, pp. 637-668.

and still result in member failures. These revisions contain the following provisions:

1. The limit in allowable nominal shear stress for members without web reinforcement shall not exceed the lesser of $0.03f_c'$ or 90 psi. This value compares reasonably well with the maximum shear stress permitted for concrete alone by the British, French, Swiss, and German codes except that the foreign codes do not have an upper limiting value. This stress also conforms to the limit used successfully in existing structures in America.

2. In addition to limiting the nominal concrete shear stress to 90 psi, continuous or restrained beams or frames, except T-beams with an integral slab, shall have web reinforcement extending from the support to a point either 1/16 of the clear span or the depth of the member, whichever is greater, beyond the extreme point of inflection even though the shearing stress in this area does not exceed allowable for concrete alone. Such reinforcement shall be designed to carry at least two-thirds of the total shear throughout this region. Web reinforcement also shall be provided sufficient to carry at least two-thirds of the total shear at any section in which there is negative reinforcement.

This provision would provide web steel in the region of failure experienced in the warehouse and would provide a means of carrying shear up to a point where a positive moment will always exist in the member. The stress condition in the portion of the member within the required web reinforcement (the portion having definite positive moment) conforms to the stress condition occurring in the many simple beam tests forming the basis of our present design codes.

Provision of web reinforcement at the ends of the member capable of carrying two-thirds of the shear at working stress will assure a factor of safety of at least 1½ even if the concrete is completely ineffective in carrying the shear. When one of the low capacity PCA test members was redesigned conforming to this provision, the factor of safety against working load exceeded 2 rather than the 1½ anticipated from the action of the web reinforcement alone and failure was initiated by yield of the main reinforceing.

Permitting the web reinforcement to be cut off at the depth of the beam or 1/16 of the span beyond the extreme point of inflection presumes that precautions will be taken in the design to limit axial tension within an amount that will maintain a flexural compression force in the area without web reinforcement. Should the axial tensile forces be large it may be necessary to extend the limits of the web reinforcement and to carefully design the top and bottom main steel.

There appears to be considerable evidence that diagonal tension type failures occur after bond slip in the bars. Careful design would accordingly suggest that adequate end anchorage should be provided by sufficient extension and/or bending of the bars beyond the point of inflection away from the area in which that bar is effective as tensile steel.

T-beam type members were specifically excluded from the requirement calling for web reinforcement beyond the point of inflection. This exclusion from the added design restrictions was made largely because of the lack of

recorded difficulties experienced in the many structures of this type built in the past. The reinforcement in the slab, reduction in both the tensile and shearing unit stresses because of participation of the flange area, and possibly the precompression caused by the more rapid early shrinkage of the thinner slab undoubtedly help to increase the shearing resistance of T-beams but further studies on this type framing are indicated as being highly desirable.

These proposed revisions, intended to prevent failures in special types of members without unduly penalizing the types of structures which have appeared to be adequate through past experience, were approved by unanimous vote at the American Concrete Institute's 52nd annual convention in Philadelphia, Feb. 21, 1956. The quantity of steel added by the proposed revisions will generally be nominal compared to the total flexural and web steel in the member and will not be changed at all for the higher unit shearing stresses (above $0.09 f_c'$). Inasmuch as the shear carrying capacity of the concrete will be ineffective in reducing the area of the web reinforcement, the tendency will be to reduce the amount of concrete and to use smaller and tougher members, with size governed largely by flexural considerations. Such proportioning is also consistent with the member proportions obtained by use of the proposed ultimate strength design methods.

Discussion of this paper should reach ACI headquarters in triplicate by May 1, 1957, for publication in the Part 2, December 1957 JOURNAL.

Title No. 53-35

Symposium on AMC warehouse failure—laboratory investigation

Laboratory Investigation of Rigid Frame Failure*

By RICHARD C. ELSTNER and EIVIND HOGNESTAD†

SYNOPSIS

An experimental investigation into the causes of failure in a rigid frame warehouse is reported. The type of diagonal cracking that caused the failure was reproduced in the laboratory; revised frame designs for future construction were tested; and remedial measures were developed to strengthen existing frames by prestressed steel strapping applied externally.

Though further studies are needed to clarify completely the fundamental mechanism of failure, it is believed that the failure took place by a combination of diagonal tension due to dead load and axial tension due to shrinkage and temperature change. It is considered highly probable that the type of distress involved can be avoided by sufficient web reinforcement, sufficient extension of longitudinal reinforcement beyond the region of contraflexure, and effective expansion joints.

INTRODUCTION

The August, 1955, collapse of a 4000 sq ft roof area at the AMC warehouse, Wilkins Air Force Depot, Ohio,[1] followed difficulties at other locations involving similar warehouse designs. An investigation by the Corps of Engineers to determine the causes of the failure immediately revealed three significant groups of facts.

First, a study of calculations and plans by the Office of the Chief of Engineers and their consultants indicated that the structural design of the warehouses was carried out in accord with commonly used minimum requirements of generally accepted American building codes.[1] The warehouses in distress had roof slabs not cast integrally with the structural frames. Many similar structures with integrally cast roof slabs have been designed by the same minimum requirements in the past, and such structures have an excellent service record.

Second, the materials used to construct the AMC warehouses were in accord with applicable specifications. The workmanship involved was in accord with common practice. Concrete cores and steel samples taken from the collapsed frames and tested by the Corps of Engineers gave strengths

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exceeding the minimum required by specifications. A complete beam section about 3 ft long and cut from a region adjacent to the failure of a frame at Wilkins was also examined by microscopic, sonic, and physical test methods at the PCA laboratories. The concrete was found to be of high and uniform quality. No abnormal behavior of cement paste or aggregates was found. Field inspections by the Corps of Engineers and consultants indicate that there was every reason to believe that neither poor foundations nor overloading caused the failure.

Third, the collapse at Wilkins originated in diagonal tension cracking of the major structural frames. This cracking was first observed about 18 months after the frames were cast. The flexural load at the time of failure was dead load of the frames and roof only.

These three groups of facts indicated that the causes for the observed distress were related to phenomena of structural strength and behavior not yet fully understood in the science of structural engineering. An experimental investigation was therefore initiated in October, 1955, at the Research and Development Laboratories of the Portland Cement Association to throw new light on these phenomena.

General objectives

The laboratory investigation reported herein has four major objectives: first, to reproduce the Wilkins failure in the laboratory; second, to test revised designs for construction of similar warehouses; third, to develop remedial measures to strengthen existing frames in distress; and, finally, to contribute to a full understanding of the basic causes for the distress experienced in the field, so that steps may be taken to insure satisfactory service of similar construction in the future.

This paper reports progress through April, 1956, when the first three objectives had been reached, and considerable advance had also been made toward the final objective. Further laboratory tests and analytical studies are needed, however, before a comprehensive and detailed understanding of the fundamental causes for the distresses can be reached.

Laboratory beam specimen

The test specimen chosen for the laboratory investigation represents a 1/3-scale model of the second frame span of the Wilkins AMC warehouse. This is the span that collapsed as indicated in Fig. 1. The test girders were subjected to 8-point loading over a 22-ft 2-in. span, and the cantilevered ends were held down in such a manner that tangents to the deflection curve at both supports remained horizontal throughout the test.

It follows from the model relationships involved that the total dead weight of a 1/3-scale model is 1/27 times that of the prototype, while the total applied loads necessary to give the same stresses in model and prototype are related in the ratio of 1:9. Hence, the uniformly distributed dead weight of the prototype frames represents a greater portion of their total working load than is the case for the 1/3-scale model. As a partial correction for this, the test

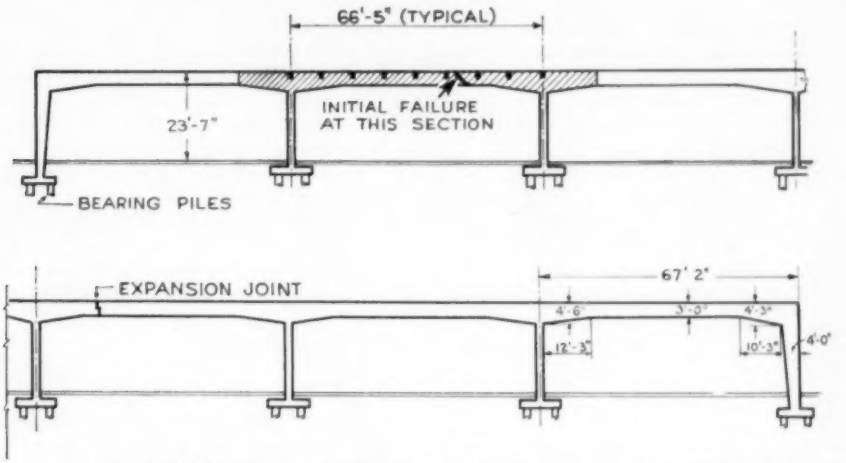


Fig. 1—Typical 400-ft frame at Wilkins depot. Shaded portions represented by laboratory specimens

beams were loaded at eight points, while the prototype was loaded by the dead weight of the roof through purlins at seven points as shown in Fig. 1. Finally, in the test beams the support tangents were held horizontal, while in the prototype the tangents may have been inclined somewhat due to the lack of symmetry.

It is seen, therefore, that the loading conditions of the test girders do not exactly represent those of the prototype. A direct comparison in terms of ultimate loads is therefore only approximate. However, the two loading conditions are so similar that the phenomena of strength and behavior observed in the test beams should apply also to the prototype. Furthermore, if such test behavior is expressed numerically in terms of stresses rather than loads, a direct comparison is possible by computing stresses for model and prototype by use of the same applicable principles of structural mechanics for both cases.

Outline of tests and results

Tests of 13 major beam specimens are reported herein as outlined in Table 1. The first seven beams were tested to failure under flexural loading only. The last six beams were subjected to combined flexure and axial tension.

The test beams are listed in Table 1 in the order that they were cast. The test results are presented and discussed by logical groups of related test beams, not by chronological order.

Notation

A_s = area of longitudinal tension reinforcement
A_s' = area of longitudinal top reinforcement
A_v = area of web reinforcement
a = eccentricity of axial load with respect to centroid of tension reinforcement
k_1k_3 = stress distribution factor of inelastic stress block
k_2 = centroid factor of inelastic stress block
M = bending moment
M_u = ultimate moment without axial load
P_a = axial tension load

b = width of beam
C = internal force in concrete compression zone
c = depth to neutral axis
d = beam depth to centroid of tension reinforcement
d' = depth to top reinforcement
f_c' = cylinder strength
f_y = yield point of reinforcement
f_s = stress in tension reinforcement
f_v = stress in web reinforcement
j = defined by internal moment arm, jd
P_f = flexural load
P_t = tie-down load
p = tension reinforcement ratio = A_s/bd
q = $p\,f_y/f_c'$
r = web reinforcement ratio = A_v/bs
s = spacing of web reinforcement
T = internal force in tension reinforcement
V = shearing force
V_1 = internal shear resistance of compression zone
V_2 = internal shear resistance of longitudinal reinforcement by dowel action

TABLE 1—OUTLINE OF TESTS AND RESULTS

Beam No.	Description	f_c', psi	Test age, days	Ultimate load, kips		v, psi	Type of failure*
				Flexure	Axial		
1	1952 design	3700	5	38.7	0	141	D.T. contraflex.
2	1952 + partial straps	2520	7	41.7	0	163	D.T. haunch
2a	1952 + full straps	2520	7	61.7	0	—	Flexure
3	1952 + reduced straps	2910	9	58.7	0	—	Flexure
4	1955 revision	2830	11	62.9	0	—	Flexure
5	1952 design	2900	53	41.2	0	151	D.T. contraflex.
6	ACI 318-56	3020	14	57.2	0	—	Flexure
7	1954 revision	3330	14	39.2	0	153	D.T. haunch
7a	1954 + partial straps	3330	14	45.7	0	167	D.T. contraflex.
8	1954 revision	4250	14	30.9	28.4	113	D.T. contraflex.
9	1952 design	3260	9	17.7	19.3	64.5	Separation
10	1952 design	3000	7	21.7	15.2	79	D.T. contraflex.
11	1952 + full straps	3290	8	61.7	10.2	—	Flexure
12	1952 + full straps	3180	7	24.7	17.6	90	Separation
13	1952 + full straps	3300	7	60.2	14.5	—	Flexure

*Denoted as flexural failure if ultimate flexural load exceeds ultimate strength calculated by limit design, which is 55.8 kips for all beams except No. 6. D.T. denotes diagonal tension failure.

MATERIALS, FABRICATION, AND TEST METHODS

The present investigation was planned, prepared for, and carried out in about 6 months. In spite of the urgent practical needs for the test results, common laboratory standards for good workmanship and precise measurements were rigidly adhered to. However, the number of measurements was held to a minimum to arrive rapidly at objectives of practical value.

Materials

The design concrete strength for the prototype frames was 3000 psi at 28 days. The basic concrete mixture used in the laboratory to produce this strength was 1:3.6:2.4. The cement used was a laboratory blend of four brands of Type I portland cement. The sand had a fineness modulus of 2.83, and the coarse aggregate was a gravel of 3/8-in. maximum size. The cement content was varied somewhat from girder to girder, and the w/c ratio by weight was varied between 0.5 and 0.6, to obtain the design concrete strength of 3000 psi at ages from 5 to 14 days by concretes with a slump from 3 to 5 in. In one case, beam 5, the cement content was reduced substantially to obtain the desired 3000 psi strength at an age of 53 days.

The longitudinal reinforcement used for the Wilkins warehouse frames was an unusually hard grade steel with a yield point of over 100,000 psi. However, an allowable steel stress of 20,000 psi was used in the frame design, and common steel grades were used in other warehouses. In the laboratory test beams, four lots of #3 intermediate-grade deformed (ASTM A 305) reinforcing bars were used as longitudinal reinforcement to simulate the #9 bars used in the prototype frames. The #3 bars used in beam 1 had a yield point of 46,400 psi, while those used in all other specimens had yield points between 48,500 and 51,000 psi.

Straightened steel wire of SWG 8 and 10, respectively, was used to simulate stirrups of deformed reinforcing bars #4 and 3 in the prototype. The 8-gage wire had an average diameter of 0.160 in. and a yield point of 30,500 psi. The 10-gage wire had an average diameter of 0.131 in. and a yield point of 36,300 psi. All wire was cleaned with a solvent and rusted by moist room storage to improve bond.

The Heavy-Duty steel straps used as external shear reinforcement in some test girders had a cross section of 0.75 x 0.035 in. Yield point of the strap was 93,500 psi, and the ultimate strength was 124,000 psi. The stress-strain curve was entirely linear to a stress of 85,000 psi with a modulus of elastiticy of 30,000,000 psi. The straps were joined by a crimped seal after obtaining a suitable strap tension as measured by electric resistance wire strain gages.

Fabrication

All beams were cast in plastic-coated plywood forms with close adherence to the dimensions given in Fig. 2. The fabrication and placing of all reinforcement cages was carried out to tolerances of less than $\frac{1}{4}$ in. Four batches of concrete were used per test beam, and three or more 6 x 12-in. test cylinders were cast from each batch. All concrete was placed by internal vibration. A 10,000 rpm vibrator with a special blade $\frac{1}{4}$ in. thick was used to place concrete through the closely spaced negative reinforcement. The concrete strengths reported for each test beam represent an average of at least 12 cylinder tests.

Test beams and the corresponding cylinders were moist cured under wet burlap for a minimum of 2 days. They were then stored in the laboratory until tested. Previous tests[2] have indicated that both the shearing and flexural strength of reinforced concrete beams depend primarily on compressive strength of the concrete at the time of beam testing, regardless of variations in curing method and age used to obtain this strength. To expedite progress of this investigation cylinder strength was adopted as the basic criterion, and age varied from 5 to 14 days except for one specimen (No. 5) which was purposely made of lower-strength concrete for testing at a later age.

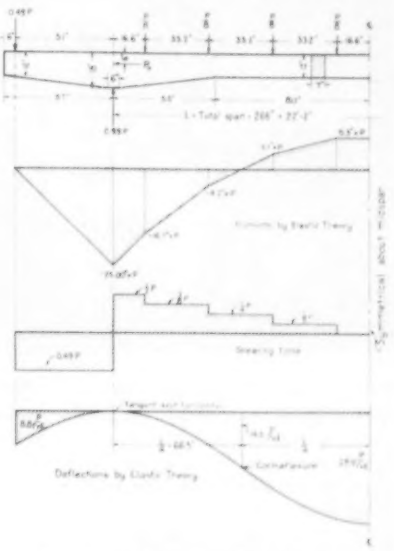


Fig. 2—Laboratory test beams

Test methods

The test beams were subjected to 8-point loading over a 22-ft 2-in. span as shown in Fig. 2. The cantilevered beam ends were held down in such a manner that tangents at the beam supports remained horizontal throughout every test.

Flexural loading was by a testing machine through a series of load-distributing beams as shown in Fig. 3. The cantilevered beam ends were held down by an independent hand-operated hydraulic jack arrangement at each end. As load was applied by the testing machine, a man at each beam end observed bench levels attached to the sides of the beam directly over the supports. By operating the jack pumps, these levels were kept in a horizontal position. The tie-down loads applied by the jacks in this manner were continuously recorded by electric strain gage recorders connected to calibrated transducers in the tie-down arrangement.

In the last six beam tests, an axial tension force was applied to the beam in addition to the flexural loading. This was done to simulate the action of volume changes, such as shrinkage and temperature drop, under restrained conditions. The tension force was applied between

supports at mid-depth of the 12-in. beam section by a hydraulic jack and a pipe column on each side of the beam acting against steel crossheads cast into the beam (Fig. 3). The two jacks were connected in series to the same oil pump. The load in each jack column was independently and continuously recorded through electric strain gage transducers.

To insure that the entire measured axial force was transmitted to the test beam, it was necessary to minimize frictional resistance to beam movements at the supports and at the eight points of flexural load application. This was accomplished by supporting the south*

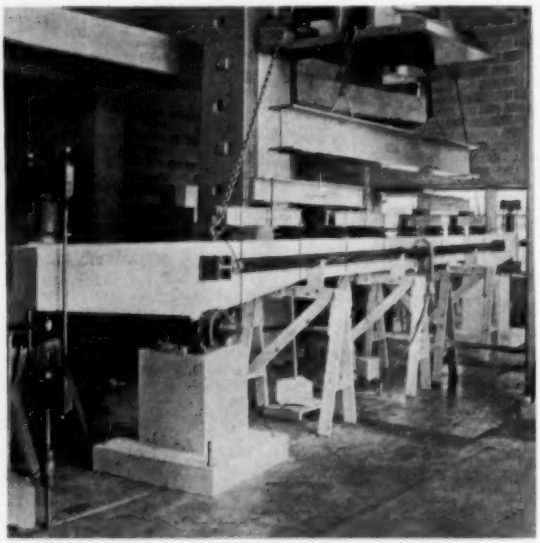


Fig. 3—Testing arrangement for beam No. 8. Steel strap to place loading frame removed during test

beam end on a 4-in. roller resting in a pair of self-centering ball bearing pillow blocks. At each of the eight flexural load points, load was applied through a half-round rocker and two bearing plates between which a ring of ball bearings allowed free horizontal movement parallel to the beam axis. The sides of the bearing plates in contact with the ball bearings were hardened and ground plane. By these loading precautions, friction was reduced to such an extent that stability of the flexural load-distributing members was maintained only by careful alignment. It is believed that practically the entire axial force, as measured by the transducers between the pipe columns and the crosshead, was actually transmitted to the test beam.

Structural analysis and properties of test beams

In addition to the geometrical proportions of the test beams and the type of loading used, Fig. 2 also gives moments, shears, and deflections as computed by the elastic theory of structures. The moments of inertia were computed for the gross concrete sections disregarding the reinforcement. Consideration was given to the geometrical shape of the haunches, which results in an increase of the support moments of 12.8 percent as compared to a prismatic beam.

In Fig. 4, a comparison is made between tie-down loads measured as a function of flexural load and the ratio $P_t/P_f = 0.49$ as obtained by the elastic theory of structures. Whenever the north and south tie-down loads coincided,

*The terms north and south refer to the position of the test beams in the testing machine. The beam end with the heaviest top reinforcement (left-hand end in Fig. 1 and 5) was always placed to the south in the machine.

only one tie-down load is shown in the figure. It is seen that the measured values follow the theoretical straight lines closely practically to failure. This is so even for beams 8 to 13 in which a considerable axial tension was introduced at the flexural load levels shown in Fig. 4. Because of this close agreement with the elastic theory, $P_t = 0.49\ P_f$ was used in all computations involved in analyzing the test data of this investigation. Also, thereby a valid comparison may be made between stresses computed from test data and results of an elastic structural analysis of the prototype frames of the AMC warehouses.

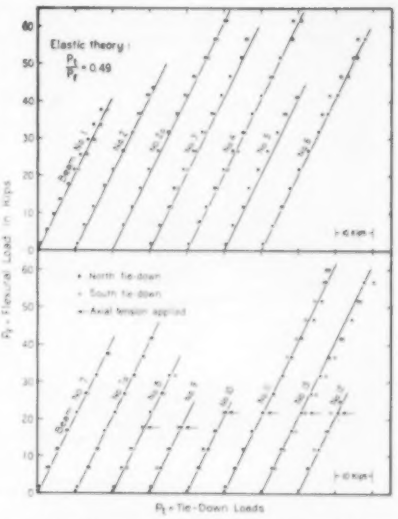


Fig. 4—Measured tie-down loads and elastic theory

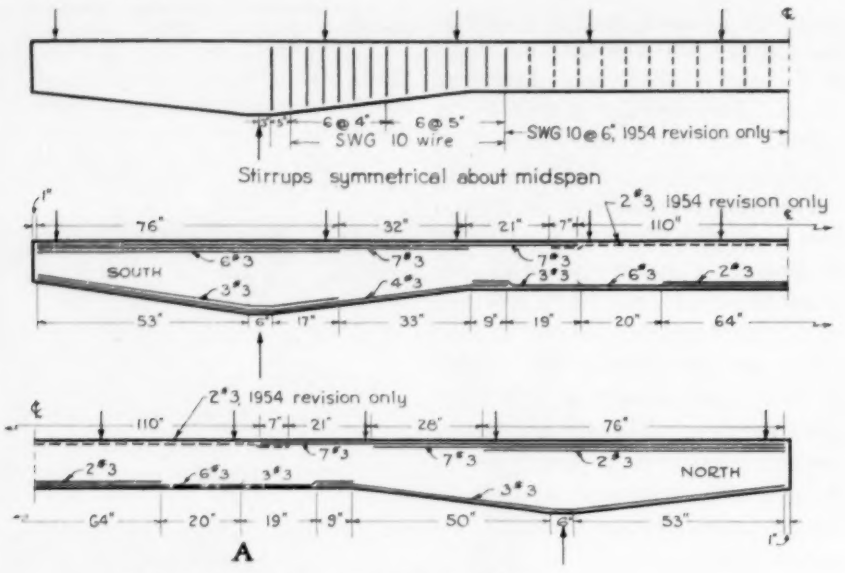
TEST RESULTS FOR 1952 DESIGN

The frames of the warehouse at Wilkins Air Force Depot were constructed in accord with a design dated April, 1952. The reinforcement used in the collapsed span of this 1952 design was reproduced in 1/3 scale as shown in Fig. 5. The south support of the test beam, over which negative reinforcement consisting of 20 #3 bars was provided, represents the first interior column (Fig. 1). The north support, with negative reinforcement of 16 #3 bars, represents the second interior column.

The 10-gage stirrup reinforcement provided was light, about 0.08 percent, and no stirrups were placed beyond the point of contraflexure. Sufficient added web reinforcement was provided in the cantilevered ends of the test beams to prevent failure in the cantilevers. No attempt was made to reproduce the reinforcement used in the corresponding parts of the prototype frames because the loading conditions of the test beams outside the supports are different from those of the prototype.

The design of the prototype frame was carried out by straight-line theory with an allowable steel stress of 20,000 psi. For the test beam loading, the loads necessary to give this calculated stress by the moment coefficients of Fig. 2 are 20.2, 20.0, and 25.3 kips for the north support section, midspan section, and south support section, respectively. The equivalent service load for the test beam is therefore 20 kips as controlled by the midspan section.

Ultimate strength was calculated for $f_y = 50{,}000$ psi and $f_c' = 3000$ psi by the procedures recommended by the ACI-ASCE joint committee on ultimate strength design.[3] Ultimate moments of 1381, 421, and 1622 kip-



Concrete cover, longitudinal reinf. = ¾-in.
Spacing between layers = ¼-in.
All web reinforcement = single closed rectangular stirrups

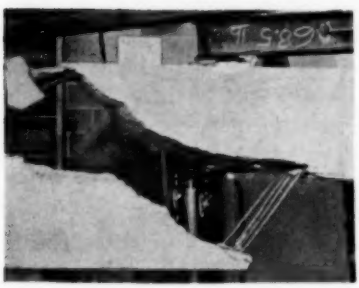
Fig. 5—Reinforcement for 1952 and 1954 designs

in. were obtained for the north support, midspan, and the south support, respectively. By the moment coefficients of Fig. 2, yielding of the reinforcement at the north support and in the midspan region should be in progress at flexural loads of 51 kips. Assuming yield hinges at both supports and at the fourth load point from the north support, an ultimate load of 55.8 kips is obtained by limit design.

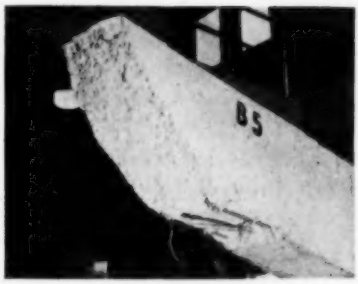
Flexural load only

Beam 1—This beam was the first one tested. The longitudinal steel used was that shown in Fig. 5 for the 1952 design. However, the prototype stirrup reinforcement was only approximated because full design details were not available to the laboratory at the time the beam was cast. Four stirrups of #2 bars were placed in each haunch at distances of 6, 12, 24, and 36 in. from the interior edge of the supports.

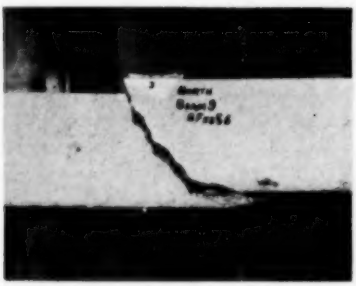
The beam was tested at an age of 5 days, at which time the average cylinder strength, f_c', was 3700 psi. Complete collapse took place by sudden formation of a diagonal crack in the contraflexure region between the second and third load from the north support at a flexural load of 38.7 kips. The crack led to complete collapse immediately after it formed as shown in Fig. 6, and a similar diagonal crack then formed after failure in the contraflexure region near the south quarter-point of the beam span. Such sudden separation has also been observed in previous laboratory tests when longitudinal



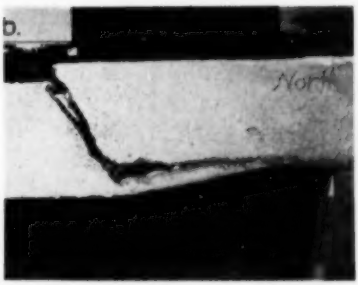
BEAM 1



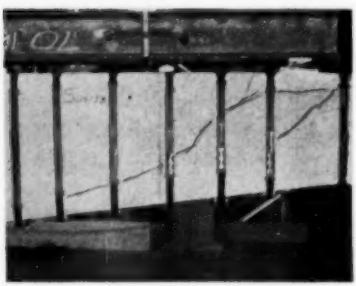
BEAM 5



BEAM 9



BEAM 10



BEAM 7
(Subsequent Strapping)

BEAM 7a

Fig. 6—Failure of 1952 and 1954 design test beams

reinforcement was terminated in the contraflexure region.[2,4]

The maximum load, P_f, at which failure took place was 38.7 kips. Taking the shear as $V = 0.25\ P_f$, the effective beam depth as $d = 11.2$ in., beam width as $b = 7$ in., and j as 7/8, gives a calculated shearing stress at failure $v = V/bjd = 141$ psi $= 0.038\ f_c'$.

Beam 5—This beam was reinforced entirely as shown in Fig. 5 for the 1952

design. The stirrup reinforcement therefore differs somewhat from that of beam 1. In the contraflexure region where failure took place, however, stirrups were entirely absent in both beams.

The beam was tested at an age of 53 days, at which time the cylinder strength was 2900 psi. Failure took place as shown in Fig. 6 in a manner similar to that described for beam 1. In both cases the diagonal crack reached the top surface of the beam close to the point where the last seven bars of negative reinforcement terminated.

The ultimate strength of beam 5 was 41.2 kips. The ultimate shearing stress was 151 psi (0.052 f_c'), as compared with 141 psi (0.038 f_c') for beam 1. Since a scatter of ± 20 percent is often associated with diagonal tension cracking loads, this difference may not be significant. However, the tests indicate that increasing age alone has no detrimental effect on the strength of these beams.

Flexural and axial load

Beams 1 and 5 failed suddenly by diagonal tension in the region of contraflexure. The location and the type of failure approximated that of the Wilkins prototype. The test beams had a region of serious shear weakness near contraflexure, since the ultimate shearing stresses of 141 and 151 psi are only about 50 percent greater than the allowable stress of 90 psi commonly permitted in design without web reinforcement.

However, the shearing stress in the prototype at the time of failure has been estimated as only 75 psi. Beams 1 and 5 therefore reproduce the Wilkins failure only qualitatively, not quantitatively.

The fact that the collapse of the prototype occurred about 18 months after casting, and the nature of other field observations, such as circumstantial evidence that the expansion joints (Fig. 1) were "frozen," indicate that shrinkage and temperature drop probably led to a development of axial tensile forces in the frame beams. Furthermore, even after 18 months the beams had not dried to equilibrium, and peaks of tensile stress existed at the surface due to differential shrinkage. No previous experimental work is reported in the literature regarding the strength in diagonal tension of reinforced concrete beams subject to combined flexural loading and axial tension. Two beams were therefore tested to explore the possibility that axial tension may reduce the ultimate strength in diagonal tension due to flexural loading.

Beams 9 and 10—These two beams were made in accord with the reinforcing details shown in Fig. 5 for the 1952 design. The average cylinder strengths were 3260 and 3000 psi, respectively, at the time of testing.

Beam 9 was initially loaded to a flexural load of 17.7 kips, which corresponds to a shearing stress of 64.5 psi (0.020 f_c') in the contraflexure region. The tangents at the supports were maintained horizontal as usual, and the midspan deflection was 0.20 in. under the flexural loading. The horizontal axial tension was then gradually applied; the flexural load was maintained at 17.7 kips, and the support tangents were kept horizontal. The average tie-down load changed only 2.4 percent from 8.33 to 8.53 kips as the axial load was

increased from zero to the failure load of 19.3 kips. This axial load of 19.3 kips divided by the gross concrete section of 12 x 7 in. gives a tensile stress of 230 psi. The midspan deflection increased from 0.20 to 0.30 in. as the axial load increased from zero to 16.0 kips.

Beam 10 was loaded in a similar manner, and the behavior under load was similar. The initial flexural load for beam 10 was 21.7 kips, corresponding to a shearing stress of 79 psi ($0.026\,f_c'$) in the contraflexure region. The beam failed at an axial load of 15.2 kips, that is, a tensile stress of 181 psi.

Failure of both beams took place violently and with little warning by a steeply inclined crack in the contraflexure region north of the third load point from the north support (Fig. 6). For both beams the crack was steeper than for comparable beams without axial load as seen by comparison of beams 5 and 10 in Fig. 6.

By arrangement with the Corps of Engineers, a 3-ft length of a beam from one of the Wilkins frames was shipped to the laboratory. The specimen included the original failure section, and a detailed imprint of the bottom longitudinal reinforcement appeared on the specimen. From this imprint it could be determined that the failure crack crossed three bottom bars about 18 in. from point A in Fig. 5, at which three out of six reinforcing bars are cut off. This corresponds to 6 in. in the 1/3-model scale, and is precisely the location of the failure crack of beam 10 as seen in Fig. 6. The inclination of the failure crack to the horizontal was about 60 deg both for the Wilkins specimen and beam 10. It is reasonable to believe, therefore, that the loading conditions that caused the Wilkins collapse are closely related to those of beam 10.

By appearance of the failure, diagonal tensile stress could have governed ultimate strength of both beams 9 and 10. However, a different hypothesis also warrants consideration. The failure section of both beams is crossed by only three longitudinal #3 reinforcing bars located near the bottom of the beam. At the top, the crack bypassed all negative reinforcement, and no stirrups are located in the failure region.

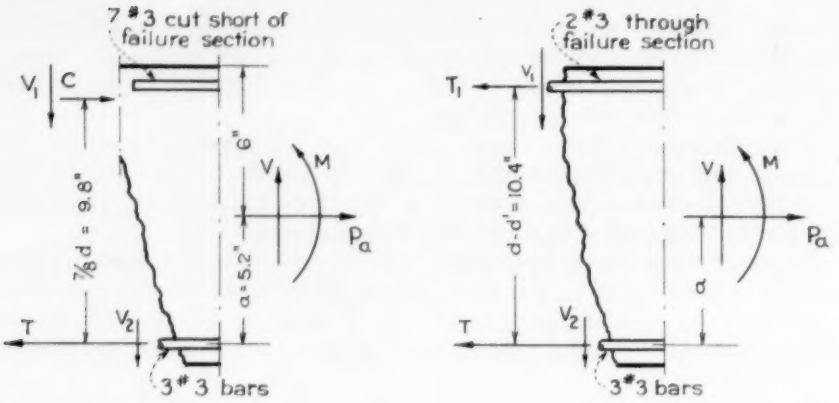
An approximate analysis may be made by considering the forces acting at the crack of beams 9 and 10 when this crack is partially developed as shown in Fig. 7a. The external forces acting are a positive moment $M = (4.1 \text{ in.})\,P_f$, an axial tension P_a applied at mid-depth, and a shearing force V. If we assume that the concrete carries no tension, the internal resisting forces are a steel tension T, a compressive force in the concrete compression zone C, a major portion of the shear V_1 carried in the compression zone, and a minor portion V_2 carried by the longitudinal steel acting as a dowel. Equilibrium of these forces gives:

$$V = V_1 + V_2 \quad (1)$$

$$P_a = T - C \quad (2)$$

$$M = aP_a = jdC \quad (3)$$

For small values of P_a, equilibrium under these conditions is entirely possible. Most of the shear is safely carried by the compression zone in the same manner as over flexural cracks in beams.



1952 design
(a) Compression zone intact

1954 design
(b) Compression zone cracked

Fig. 7—Equilibrium in contraflexure region with axial load

If an initial moment M is applied, and the axial load is subsequently increased, then Eq. (3) indicates that C must decrease with increasing P_a. That is, the crack will progress upward. At a value of P_a given by

$$P_a = \frac{M}{a} \qquad (4)$$

the force in the "compression" zone becomes a tension. If the crack then progresses to the top of the beam as P_a is increased beyond M/a, and there is no top reinforcement, the crack will open up to give enough longitudinal beam movement to relieve P_a. After such separation, the entire shear force V is transferred to the bottom steel, $V_2 = V$, and the beam fails by stripping out the reinforcement. Such stripping took place in beams 9 and 10 (Fig. 6). The type of failure described takes place by separation, and the value of P_a given by Eq. (4) may therefore be referred to as the separation load. Equilibrium for the condition $P_a > M/a$ can be maintained only by tensile stress in the concrete.

Consider a different mechanism of failure. If initially a working load moment equal to about one-half of the ultimate moment in pure flexure is applied, and the axial load P_a is subsequently increased, yielding of the longitudinal reinforcement will be initiated at a load found by solving Eq. (2) and (3) for $T = A_s f_y$:

$$P_a = \frac{A_s f_y jd - M}{jd - a} \qquad (5)$$

Rather than take an approximate value of $j = 7/8$ or 1.0, a rigorous solu-

tion was obtained by the methods of inelastic flexural analysis.[5] For $C = k_1k_3\,bc\,f_c'$, $T = A_sf_y$, $jd = d - k_2c$, and $k_2/k_1k_3 = 0.59$, we obtain

$$P_a = \frac{f_c'\,bd}{1.18}\left[(1.18q + \frac{a}{d} - 1) + \sqrt{(1.18q + \frac{a}{d} - 1)^2 + 2.36\,\frac{M_u - M}{bd^2\,f_c'}}\right] \ldots (6)$$

in which M_u is the ultimate flexural strength without axial load [$M_u = bd^2\,f_c'\,q\,(1 - 0.59q)$], M is the applied flexural moment, and $q = p\,f_y/f_c'$.

For the failure section of beams 9 and 10 at a moment $M = 21.7 \times 4.1$ kip-in., Eq. (5) gives $P_a = 15.9$ kips for $j = 7/8$, and $P_a = 16.0$ kips for $j = 1.0$. Eq. (6) gives $P_a = 16.0$ kips, and the corresponding depth to the neutral axis is $c = 0.09$ in. A value $j = 1.0$ is therefore used for analysis of test data.

The shallow depth to the neutral axis gives an ultimate steel strain of over 0.30 for an ultimate concrete strain of 0.003. Such a strain of 0.30 would obviously be beyond the ultimate strength of the reinforcement used. Accordingly, yielding of the bottom reinforcement at the yield load value of P_a given by Eq. (5) or (6) need not cause failure in flexure. It may be possible by strain hardening to develop the full ultimate strength of the reinforcement. On the other hand, failure by diagonal tension may be initiated by yielding of the reinforcement. A shearing off of the compression zone due to the force V_1 (Fig. 7) followed by stripping of the reinforcement is also probable before the depth of the compression zone becomes as narrow as 0.09 in.

Consider then the failures of beams 9 and 10 in terms of the separation load and yield load as given by Eq. (4) and (5). Failure of beam 9 took place at a flexural load of 17.7 kips and an axial load of 19.3 kips. The corresponding separation load is 14.0 kips. The yield load of 18.7 kips by Eq. (5) is not valid even theoretically because it exceeds the separation load, so that in this case, the yield load is the full yield force of the reinforcement of 16.5 kips. Beam 10 failed at 21.7 kips flexural load and an axial load of 15.2 kips. The separation and yield loads are 17.1 and 16.0 kips. These figures indicate that beam 9 failed by separation and yielding of the longitudinal reinforcement, while beam 10 failed by diagonal tension.

There are two consequences of such a hypothesis regarding the mode of failure of beams 9 and 10. First, if failure of beam 9 is due to separation or yielding of the tension reinforcement, then the addition of vertical web reinforcement to a similar beam should not change the ultimate load combination when the axial force at failure exceeds 16.5 kips. Secondly, a similar beam should be strengthened substantially for an axial force exceeding 16.5 kips by addition of top reinforcement crossing the failure crack so that separation is prevented. Both of these consequences were confirmed by later tests of beams 12 and 8. If beam 10 failed by diagonal tension, on the other hand, a similar beam with an axial load less than the separation and the yield load should be strengthened by web reinforcement. The later test of beam 13 showed that for an axial load of 14.5 kips, the flexural ultimate load was in-

creased almost 200 percent as compared to beam 10 by addition of web reinforcement.

It is a fact that the ultimate flexural loads of beams 9 and 10 (17.7 and 21.7 kips) were substantially reduced as compared to beams 1 and 5 (38.7 and 41.2 kips) by application of axial loads. It seems probable that failure of beam 9 was initiated by yielding of the longitudinal reinforcement and separation at the failure section, while beam 10 failed initially by diagonal tension. Future tests of similar beams with axial loads in the range from zero to 15 kips, and with strain gages attached to the longitudinal bars at the critical section near the third load point, are needed to clarify the mode of failure.

Summary

The tests of beams 1, 5, 9, and 10 indicate that the contraflexure region of the 1952 design tends to be weak in two respects. First, with only flexural loads applied, diagonal tension failure is probable at a calculated shearing stress ($v = V/bjd$) equal to about 150 psi. Secondly, a section located close to the point of contraflexure is crossed by only three #3 reinforcing bars (three #9 bars in the Wilkins prototype).

For axial tensions of about 200 psi, failure is probable at a calculated shearing stress due to flexural loading of about 70 psi. Further tests are needed to investigate the extent to which interpolation is valid between these cases. For instance, for an axial tension of 100 psi due to shrinkage and temperature drop, is failure probable at a calculated flexural shearing stress of about 110 psi?

It is believed that stresses computed by analytical studies of the prototype frames may be compared to the stress conditions above. Stresses in the prototype should then be estimated by commonly used elastic methods of frame analysis, and moments of inertia as well as section areas used should be those of the gross concrete sections.

TEST RESULTS FOR 1954 AND 1955 DESIGNS

1954 revision of 1952 design

As a result of experience during construction, the Office of the Chief of Engineers revised the 1952 design in March, 1954. Top bars and nominal stirrups (0.06 percent) were provided for the full length of the frames. Several warehouses similar to that at Wilkins Air Force Base were subsequently constructed at various locations in accord with this 1954 revised design. The revisions provided for in the laboratory test beams are shown as dotted lines in Fig. 5.

Beams 7 and 7a—Beam 7 was made in accord with the 1954 revised design as shown in Fig. 5. The average cylinder strength was 3330 psi at the time the beam was tested. The beam failed by diagonal tension in the south haunch at a flexural load of 39.2 kips. The diagonal crack shown for beam 7 in Fig. 6 (photographed after strengthening by steel straps) developed slowly,

but collapse appeared to be imminent when the load was removed. By comparing the ultimate flexural load of 39.2 kips to the loads of 38.7 and 41.2 kips of beams 1 and 5 of the 1952 design, it is seen that the 1954 revision did not increase the ultimate load for flexural loading without axial load, though the location of failure changed.

Beam 7 was unloaded, strengthened at the haunch by externally applied straps, and reloaded to a second failure as test beam 7a. The strapping operation is discussed later in this paper. Failure took place directly under the third load point from the south support as shown for beam 7a in Fig. 6 at a flexural load of 45.7 kips. The calculated shearing stress was 167 psi ($0.050\ f_c'$), using shearing force to the left of the load point. The nominal stirrup reinforcement provided in the region of contraflexure increased the diagonal tension strength only slightly over the value of 151 psi obtained for beam 5, although in the laboratory beam it transferred the weakest region to the haunch.

Beam 8—This beam was subjected to combined axial and flexural load. The reinforcement used was identical to that of beam 7. The average cylinder strength was 4250 psi.

The beam was initially loaded in flexure to 17.7 kips, and the axial load was then gradually increased to 20 kips. Under the same flexural load of 17.7 kips, beam 9 of the 1952 design failed at an axial load of 19.3 kips. Beam 8 showed no signs of distress at the load combination of 17.7 and 20.0 kips. The flexural load was then increased to 21.7 kips, and the axial and flexural loads were subsequently alternately increased in increments of about 2 kips. At a flexural load of 24.2 kips and an axial load of 24.0 kips, an inclined crack developed in the contraflexure region near the third load point from the north support. This crack widened gradually as loading continued, and failure took place at a flexural load of 30.9 kips ($v = 113$ psi $= 0.027\ f_c'$) combined with an axial load of 28.4 kips. The appearance and location of the failure was similar to that shown for beam 10 in Fig. 6.

The failure crack of beam 8 is crossed by three #3 bars at the bottom and two #3 bars at the top as shown in Fig. 7b. The force corresponding to the 50,000 psi yield point of these five bars is 27.5 kips. This again indicates that yielding of the longitudinal reinforcement, or diagonal tension stress, or a combination thereof could be the primary cause of failure. Further tests at intermediate axial load levels and with suitable instrumentation are necessary to clarify the mode of failure.

The stress in the bottom steel at failure may be computed by rearranging Eq. (5) for $j = 1.0$:

$$A_s f_s = \frac{P_a (d - a) + M}{d} \qquad (7)$$

Considering that a increased to 6.0 in. owing to beam deflection before failure took place, a steel stress of 84,000 psi is obtained, which exceeds the ultimate strength of 78,000 psi. The steel did not fracture, and it is therefore probable

that the point of contraflexure moved closer toward the failure crack after the steel yielded so that the moment M was decreased. Such motion of the point of contraflexure was indicated also by strain gage measurements.

The separation condition may be investigated by considering the top reinforcement yielding after the crack extends entirely through the beam (Fig. 7b). Eq. (4) will then take the following form:

$$P_a = \frac{M + (d - d') A_s' f_y}{a} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

For $a = 6$ in., Eq. (8) gives $P_a = 35.0$ kips. This indicates that separation of the beam was not possible.

The ultimate calculated shearing stress of beam 8 was reduced by the axial load to 113 psi as compared to 153 and 167 psi for beams 7 and 7a in spite of the higher concrete strength for beam 8.

Compared to beams 9 and 10 of the 1952 design, beam 8 sustained increases of about 50 percent in both flexural and axial load. It should be noted, however, that the diagonal tension failure in the haunch of beam 7 of the 1954 design, subject to flexural loading only, took place at a slightly *smaller* load than that of beam 5 of the 1952 design. The increase in strength of beam 8 under the combined loading condition shows that the 1954 design may be superior when axial tension is present. However, the higher concrete strength may have contributed to the greater ultimate strength of the laboratory beam. A more complete understanding of the strength of this redesign must await tests with intermediate combinations of axial and flexural load.

1955 revision of 1952 design

Following the Wilkins collapse, a new design was prepared under the direction of the Office of the Chief of Engineers for further construction of similar warehouses. To minimize the cost of strengthening the frame design for those cases where the reinforcing steel had already been fabricated, a design was prepared in October, 1955, to utilize the longitudinal reinforcement of the 1954 design as much as possible. The details of this redesign have been described elsewhere.[1] As it applies to the second frame span, the revisions involved were reproduced in detail in 1/3 scale as laboratory test beam 4.

Beam 4—The longitudinal reinforcement of the test beam was identical to the 1952 design shown in Fig. 5 with two exceptions. A top reinforcement of five #3 bars was provided, and the splices of these bars to the negative reinforcement extending from both supports were staggered over a length of 23 in. Furthermore, two continuous #2 bars intended to control local cracking were added on each vertical face of the beam.

The stirrup reinforcement of this 1955 revision was greatly increased to carry the entire shear in the region of low moment without any aid from the

concrete. The ratio of stirrup reinforcement in the contraflexure region of the test beam was increased to 0.60 percent as compared to 0.06 percent for the 1954 revision. The minimum stirrup reinforcement provided at midspan was 0.18 percent, and a maximum of 0.67 percent was provided close to the supports.

The beam was tested with flexural loading only when the average cylinder strength was 2830 psi. Before the maximum load of 62.9 kips was reached, electric strain gages attached to the reinforcement indicated that the longitudinal reinforcement was yielding at both supports and in the midspan region. Crushing of the concrete at the bottom of the beam near the north support began at a load of 62.2 kips. Even so, the load subsequently increased to 62.9 kips, and the beam continued to deform while maintaining this load until the test was discontinued at a midspan deflection of 7 in. No signs of diagonal tension distress were found at any stage of the test. Diagonal cracks began to develop at a load of 36.7 kips, but the cracks were entirely confined by the stirrups.

This test strongly indicates that ample shearing strength is provided by the 1955 revision. The ultimate beam strength of 62.9 kips exceeds by 12.5 percent the ultimate strength of 55.8 kips calculated by limit design, so that strain hardening of the longitudinal reinforcement was probably in progress when it was considered that the beam had failed by excessive deflection. Furthermore, no beam section has less longitudinal reinforcement than eight #3 bars plus the four #2 face bars. The axial force required to yield this reinforcement is about 50 kips as compared to 16.5 kips for the original 1952 design. It is highly probable, therefore, that the 1955 revision also provides ample resistance to axial forces that may develop due to shrinkage and temperature change.

Design by 1956 ACI Building Code

The provisions of the ACI Building Code pertaining to shearing strength of free standing frames were improved in the 1956 code revision (ACI 318-56). Test beam 6 was designed by the writers in accord with the new provisions. Unlike the other beams of this investigation, the longitudinal and web reinforcement of beam 6 were designed for the exact loading conditions of the laboratory tests. Beam 6 is therefore purely a laboratory design.

Beam 6—The design was made for a working load per foot of length equal to 1/3 of that for the 1952 design of the Wilkins frames, namely a total flexural load of 22.5 kips. This corresponds to 2.82 kips for each of the eight concentrated loads as outlined in Fig. 8.

The longitudinal reinforcement is similar to that of the 1952 design, though the negative reinforcement is 18 #3 bars at both supports, and the positive reinforcement was increased to nine #3 bars. A major change, however, is the extension of six #3 bars of the negative reinforcement a distance 1/16 of the clear span beyond the point of contraflexure in accord with ACI 318-56, Section 902(a).

The stirrup reinforcement was increased considerably as compared with

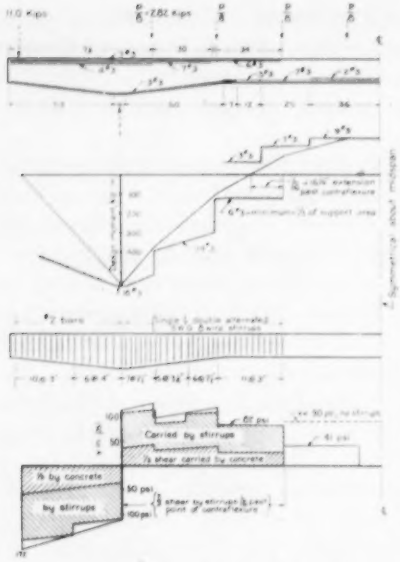


Fig. 8—Design by ACI 318-56, beam No. 6

the 1952 design. To comply with ACI 318-56, Section 801(e), the stirrups were designed to carry 2/3 of the shear from the supports to a point 1/16 of the clear span beyond the point of contraflexure. Thereby the web reinforcement ratio was increased as compared with the 1952 design from 0.10 to 0.34 percent at the support, and from 0 to 0.28 percent in the contraflexure region. It is seen in Fig. 8 that the stirrup reinforcement actually provided in the test beam was slightly less than that needed to carry 2/3 of the shear. This is because stirrup spacing was taken to the nearest ¼ in. and the SWG 8 wire actually used had an area slightly below the nominal area corresponding to that gage.

The beam was tested at a concrete strength of 3020 psi. Failure took place as shown in Fig. 9 between the third and fourth load points from the north support at an ultimate flexural load of 57.2 kips. The shear in this region is 1/8 of the flexural load, so that the shearing stress at failure was 104 psi (0.035 f_c'). The design shearing stress (Fig. 8) was 41 psi, which is well below the value of 90 psi permitted on the concrete of an unreinforced web according to ACI 318-56, Table 305(a).

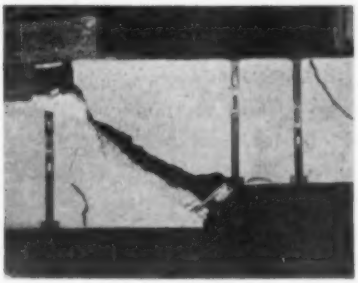
Because the longitudinal reinforcement of the beam was designed for the test loading conditions, first yielding at both supports and at midspan, as calculated by inelastic theory, practically coincides with the ultimate strength of 56.7 kips calculated by limit design. At the observed ultimate load of 57.2 kips strain gages registered a steel strain 10 percent below yielding at midspan and 5 percent over yielding at the north support. It is probable, therefore, that the failure seen in Fig. 9 was initiated by yielding of the bottom steel, and it is important to note that the ultimate strength in flexure calculated by limit design was exceeded.

LABORATORY TESTS OF REMEDIAL MEASURES

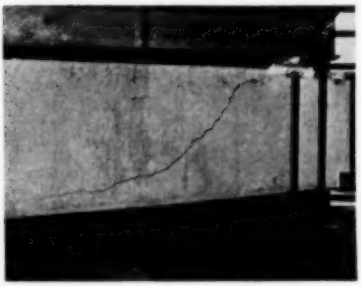
The collapse at Wilkins Air Force Base and the laboratory tests reported herein strongly indicated a diagonal tension weakness in the contraflexure region of the 1952 frame design. Laboratory studies of remedial measures therefore became an important part of this investigation, and field tests reported elsewhere were carried out by the Corps of Engineers.[6]

The first method considered was the use of steel yokes placed on top and

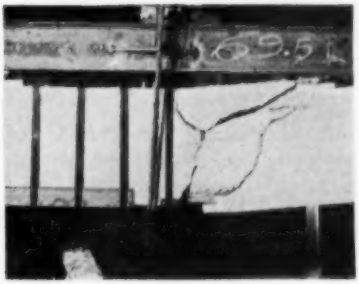
BEAM 6



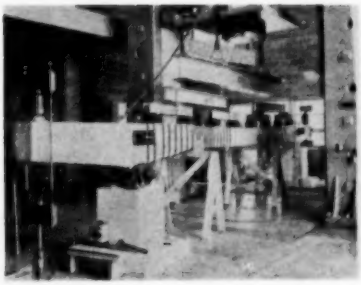
BEAM 3



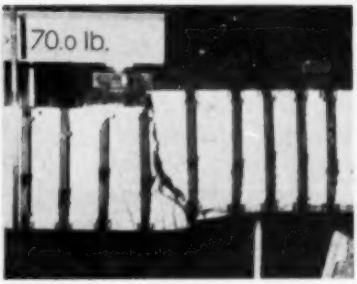
BEAM 2



BEAM 2a



BEAM 11



BEAM 12

Fig. 9—Failures of redesigned and strapped beams

bottom of the beam and connected by vertical bolts on the two beam sides. This method has been used rather extensively both in this country and abroad in laboratory work. The purpose in the laboratory has generally been to force a flexural failure in cases where it is desirable to study flexural strength and behavior without influences from shear failures. It has been clearly shown in such work that bolted "outside stirrups" constitute a reliable method of increasing the diagonal tension resistance of reinforced

concrete beams. In the present practical case, however, such bolted yokes would be relatively expensive, and in some cases it would be necessary to cut through the roof to place yokes on the top of beams.

The second method considered was the use of the steel strapping widely used in material handling. Steel strap and corresponding tightening and sealing equipment large enough to strengthen the prototype warehouse frames are commercially available. This method requires practically no headroom over beams, and it was also considered economical. All laboratory tests of remedial measures therefore involved only the use of steel strap as a means of increasing diagonal tension resistance.

Preliminary tests

No information was available regarding the action of steel strap as web reinforcement for reinforced concrete beams. It seemed probable that straps would perform this function as well as bolted yokes, but a laboratory investigation was needed to verify this.

Simple beams—Ten beams with a cross section of 6 x 12 in. were loaded to failure at third-points over a 78-in. span. The average concrete strength was 4300 psi, and two #8 bars with a yield point of 47,000 psi were used as longitudinal reinforcement in all beams.

The first beam was tested to failure without external straps. It failed in diagonal tension at an ultimate load of 26.2 kips. The inclination of the diagonal crack that caused failure was about 45 deg. The shearing stress at failure was 240 psi.

Strap reinforcement was then designed as vertical stirrups by the equation $A_s = Vs/f_s jd$. The shear, V, was taken as that corresponding to the ultimate strength in flexure minus the shear at failure of the beam without straps. A stress f_s of 40,000 psi was used. On this basis, a reinforcement of 14 straps at each beam end was arrived at, using the smallest Heavy-Duty strap available, 0.75 x 0.035 in. These straps were applied with a final prestress of 50,000 psi as measured by SR-4 gages after sealing. At a load of 37 kips, diagonal cracks inclined about 30 deg formed. The load was then increased to the calculated flexural strength of 50 kips. There was no evidence of shear failure, and the load was removed.

Half of the straps at each beam end were then removed and the beam was again loaded to 50 kips without any sign of shear failure. After unloading again, and removing three more straps at each end, the beam finally failed with only four straps at each end at a load of 48.8 kips, and without breaking the straps. The computed strap stress at failure was 160,000 psi, which exceeds the ultimate strength of 124,000 psi of the strap used. For the test beam working load of 20 kips, the shearing stress was 180 psi. If the concrete is assumed to carry 90 psi, four straps at each end give a strap working stress as high as 67,000 psi. It appeared, therefore, that the use of the truss analogy equations of the ACI Building Code to design prestressed strap reinforcement should result in a conservative and safe design.

The remaining eight simple beams were used to study the effect of strap

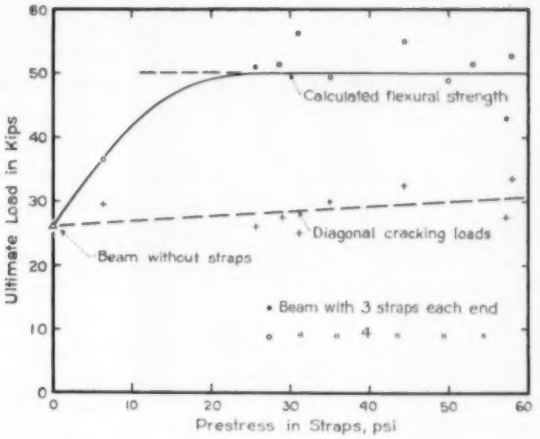


Fig. 10—Tests of simple beams, strap prestress varied

prestress on the strength of the beams. Strap prestress was measured by electric strain gages. As shown by Fig. 10, the diagonal cracking load increased somewhat with increasing prestress. All beams except one failed in flexure, or in shear at loads close to the flexural capacity, so that the longitudinal reinforcement was probably yielding at failure. The behavior of the beam with straps prestressed to only 6300 psi differed from that of the remaining beams. A diagonal crack opened abruptly at a load of 29.5 kips. As further load was applied, the straps were unable to keep the diagonal cracks well closed, and the beam failed at the relatively low load of 36.5 kips.

The test data given in Fig. 10 indicate that, to be fully effective, the straps should be sufficiently prestressed to fit the beam snugly. At diagonal cracking, shear force is then transferred to the straps before the cracks open to such an extent that the compression zone at the top of the crack fails by crushing or tearing of the concrete. A prestress as low as 25,000 psi seems to insure full efficiency of the straps as shear reinforcement.

Tests of full-size strap—Preliminary tests were made to determine whether 2.0 x 0.050-in. strap, the largest size available, can be prestressed satisfactorily for use as shear reinforcement. The strap used had a yield point of 80,000 psi and an ultimate strength of 120,000 psi. The seals were of the friction type. Stretching and sealing equipment was made available by the Signode Steel Strapping Co. The worm-gear drive stretcher Model W2 was found satisfactory for one-man operation.

Stretching and sealing were performed on the 20-in. dimension of a 20 x 22-in. reinforced concrete column. The first strap was applied without protection or lubrication at the column corners. Electric strain gages attached to the strap indicated that the prestress obtained at the column side opposite the stretcher was only 19,000 psi, while stresses at the two sides adjacent to the stretcher were 42,000 and 35,000 psi. It was difficult to remove the stretcher after the strap was sealed, and considerable prestress was lost. The unprotected corners of the column were not damaged by the strapping.

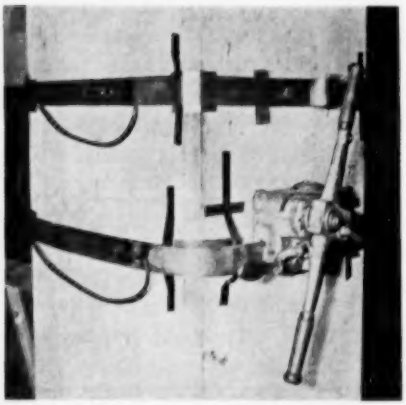


Fig. 11—Stretching test of 2.0 x 0.05-in. strap

A second strap was then applied as shown in Fig. 11. To reduce friction, steel strips lubricated with Molykote Type G were placed at the column corners. Furthermore, ¾-in. shims were placed under the strap on each side of the stretcher to facilitate its removal after sealing the strap. In this manner a final prestress of 45,000 psi was obtained opposite the stretcher, and the stress on both sides adjacent to the stretcher was 46,000 psi. The stress in the strap decreased about 700 psi during one week, and most of this loss took place during the first 24 hr. By cutting the strap after 6½ months and measuring the release strain, a remaining average prestress of 42,800 psi was found.

Summary—Though much can yet be learned by simple beam tests regarding the action of steel strap as shear reinforcement, the following conclusions seem acceptable: (a) Design of steel strap as web reinforcement by the truss analogy methods of the ACI Building Code should lead to conservative designs even if a shearing stress of 90 psi is assumed carried by the concrete. (b) The straps are fully effective even with a prestress as low as 25,000 psi. (c) It is possible to prestress a 2.0 x 0.050-in. strap to an effective prestress of at least 45,000 psi, and to retain 95 percent of this prestress after 6 months.

Beams with flexural load only

Steel strap with a cross section of 2.0 x 0.050 in. is a practical size for strengthening the prototype frames of the AMC warehouses. The area of this strap is 0.10 sq in., so that the corresponding strap for a 1/3-scale model should have an area of 0.10/9 = 0.011 sq in. However, the smallest size strap available in the Heavy-Duty quality (high yield point) is 0.75 x 0.035 in. and has an area of 0.026 sq in. The test beams were therefore strapped as indicated in Fig. 12. To simulate remedial measures considered by consultants to the Corps of Engineers, 0.75 x 0.035-in. strap was used at such spacing that the strap area per unit length of the test beams was 1/9 that for the prototype; that is, the web reinforcement ratio was the same for model and prototype.

Beams 2 and 2a—Embedded longitudinal and stirrup reinforcement of the 1952 design (Fig. 5) was used in beam 2. Testing took place at an age of 7 days when the average cylinder strength was 2520 psi. The steel straps used for this beam, shown in Fig. 12, simulate a design for the prototype that was under consideration at the time the beam was made. The ratio of strap reinforcement in the contraflexure region was 0.25 percent. For the test beams this ratio corresponds to a shearing stress of 234 psi carried by the straps alone at their yield point of 93,500 psi. That is, the straps alone at their

yield point carry the entire shear corresponding to a flexural load of 64 kips, which slightly exceeds the full flexural strength of the test beam.

The straps were initially applied with a prestress of 55,700 psi as measured by electric strain gages. However, owing to uneven contact between the concrete beam and the external steel angles used, the prestress tended to drop as succeeding straps were applied. The final average prestress at the time the beam was tested was 46,800 psi.

At a flexural load of 32.0 kips, diagonal cracks formed in the unstrapped haunches, and these cracks widened as loading was continued as shown in Fig. 9. At a load of 41.7 kips, the width of the cracks indicated that the light embedded stirrup reinforcement was yielding, and that the beam was in serious distress. The beam was therefore unloaded before complete collapse took place.

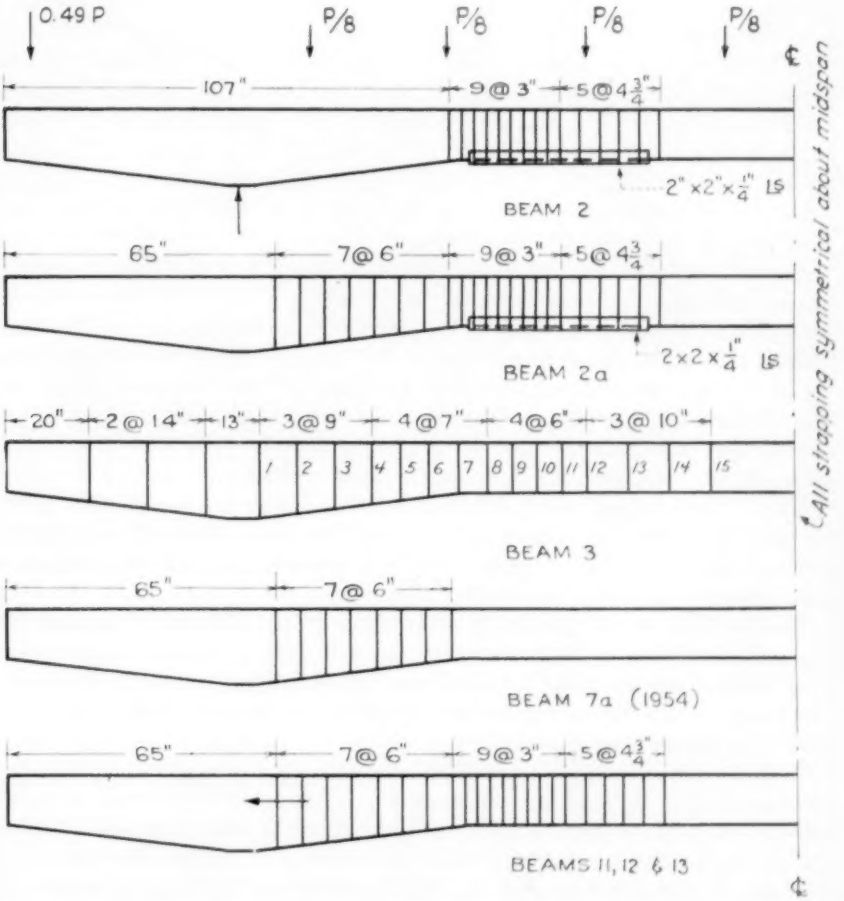


Fig. 12—Strapping of test beams. All straps were 0.75 x 0.035-in. Heavy-Duty strap

The computed shearing stress for the test beam at this load of 41.7 kips is 163 psi (0.065 f_c'). Subtracting the contribution of the stirrups at their yield point stress, shearing stress carried by concrete was calculated as 123 psi (0.049 f_c'). No diagonal cracks formed in the strapped contraflexure region.

After the load was removed, the width of the diagonal crack in the north haunch was 0.09 in. as measured by a graduated microscope. External straps were then applied to the haunches as shown for beam 2a in Fig. 12. The strapping closed the north diagonal crack to 0.06 in. The beam was then reloaded and designated as beam 2a.

When the beam was reloaded, the straps in the haunches restrained the diagonal cracks from opening. At a load of 53.7 kips an inclined crack formed in the unstrapped region on the north side of the first load point north of midspan. The calculated load at which yielding of the longitudinal reinforcement should start was 51.0 kips. This crack widened gradually until the beam failed at a load of 61.7 kips in the manner shown for beam 2a in Fig. 9. This load is 10 percent over the flexural capacity of 55.8 kips calculated by limit design. The beam failed somewhat violently but with adequate warning. Strain gage readings indicated yielding of the longitudinal reinforcement at both supports and at midspan before failure took place. Immediately prior to the final failure, no diagonal cracks had formed in the originally strapped contraflexure region, and the cracks in the haunches remained closed by the subsequently placed straps.

Beams 7 and 7a—As described previously, beam 7 of the 1954 design was tested without strap reinforcement, and reached a condition of serious distress due to diagonal cracking in the haunch at a flexural load of 39.2 kips. This was nearly the same as the load of 41.7 kips, which caused serious cracking of beam 2 before the haunches were strapped, and confirms the finding that the ultimate calculated shearing stress for the haunches of the 1952 and 1954 design test beams is only about 120 psi (0.04 f_c') plus a contribution of about 40 psi (calculated as $r f_v$) by the embedded stirrups.

Beam 7 was subsequently strapped only in the haunch region as shown for beam 7a in Fig. 12. The straps restrained the diagonal cracks from further opening under increased loading until failure took place in the unstrapped contraflexure region at a load of 45.7 kips.

The tests of beams 2a and 7a show clearly that externally applied prestressed straps can be used effectively as web reinforcement. Straps placed after diagonal cracking had developed prevented further opening of the cracks, and straps placed before the beams were loaded prevented diagonal cracking in the contraflexure region.

Beam 3—The embedded longitudinal and stirrup reinforcement of this beam was that of the 1952 design. The strap reinforcement was designed by the writers to be light enough to result in failure of the straps during beam testing. The number of external straps was reduced from 22 as used for beam 2 at each end of the beam span to 15, and the final average strap prestress was reduced from 46,800 psi for beam 2 to 36,000 psi for beam 3.

The beam was tested at an age of 9 days when the average cylinder strength was 2910 psi. Strains were measured in all straps by electric strain gages. In Fig. 12 the straps are numbered from 1 to 15 for the south half of the beam in which failure took place.

Diagonal cracks formed in both haunches at a load of 32.0 kips, the load at which the unstrapped haunches of beam 2 also cracked. At a load of 47.0 kips, a diagonal crack formed between the third and fourth load from the north support. These three diagonal cracks were prevented from opening by the straps, and did not cause failure. Failure took place abruptly by formation of a new diagonal crack in the contraflexure region between the second and third load from the south support, and fracture of straps 9 and 10 followed immediately as shown in Fig. 9 at a load of 58.7 kips.

The strain measurements indicated that the stress in all straps remained at the prestress level until the haunches cracked at 32.0 kips. At a load of 56.7 kips the stresses in straps 5, 6, and 7 increased by 9300, 17,400, and 16,500 psi, respectively. Straps 1 and 2 became active at 40.0 kips when inclined flexural cracks developed near the south support; the stress increases were 9600 and 21,300 psi at a load of 56.7 kips. At this load the stress increases in all other straps, including straps 9 and 10, were less than 5000 psi.

The failure load of 58.7 kips is 5 percent above the ultimate flexural strength of 55.8 kips calculated by limit design. Yield point strains were recorded for the reinforcement over the north support before failure, and 90 percent of the yield point was indicated at midspan. The strap spacing in the contraflexure region where beam 3 failed was 6 in., and the ratio of strap reinforcement was 0.125 percent. The shear carried by the strap at the yield point corresponds to a flexural load $P_f = 4V = 4A_v\, jd\, f_y/s = 32.0$ kips, which is 55 percent of the observed strength of 58.7 kips.

Summary—From the tests of strapped beams it may be concluded that strap reinforcement designed to carry half of the shear corresponding to ultimate flexural strength at the strap yield point should just suffice to develop the flexural strength (beam 3). Assuming a safety factor of 2.0 for flexure, this conclusion may be restated as: strap reinforcement designed to carry the full working load shear at the yield point stress of the strap should suffice.

Accordingly, it seems reasonable to design the straps to carry the entire working load shear at a strap stress of 50,000 to 60,000 psi. It should be made unmistakably clear, however, that it is not satisfactory to design the strap to carry the excess over 90 psi considered as carried by the concrete. Such a design may result in absence of straps in the contraflexure region.

Flexural and axial load

The laboratory test without strapping of beams of the 1952 design indicated that the shearing stress at failure for flexural loading only was about 150 psi. The dead load shearing stress at which the Wilkins collapse took place has been estimated as about 75 psi due to flexural loading alone. The tests of

beams 9 and 10 indicated that the shearing strength of the 1952 design beams is reduced when an axial load is combined with the flexural load, and it is highly probable that the Wilkins failure was caused by a combination of flexural and axial stress.

Though beams 2a and 3 were strengthened satisfactorily by steel strapping, so that they exceeded their calculated flexural capacity when tested under flexural loading only, it was considered necessary to investigate also the effect of axial tension on beams strengthened by strapping. Three beams were tested, all with internal stirrups and longitudinal reinforcement in accord with the 1952 design. The straps were all applied before the test started at the locations shown for beams 11 to 13 in Fig. 12. The prestress level of 40,000 to 50,000 psi was essentially judged by the operator of the strapping device. Previous strapping operations in which electric strain gages were used to measure the prestress indicated that the judgment of the operator is remarkably reliable in this case. The strap locations were the same as those of beam 2a, but the steel angles in the contraflexure region were omitted.

Beam 11—This beam was tested at an average cylinder strength of 3290 psi. The beam was first loaded to a flexural load of 21.7 kips. The axial load was then gradually increased from zero to 10.2 kips while the flexural load was maintained. Finally, the flexural load was increased to failure while the axial load was maintained at 10.2 kips. Throughout the test, the tie-down loads at both beam ends were adjusted to keep both support tangents horizontal.

Diagonal cracks appeared in both contraflexure regions at a flexural load of 26.7 kips plus the axial load. A diagonal crack under the third load point from the north support formed at a load of 46.7 kips, followed by diagonal cracking close to the north support at 55 kips. The beam failed in flexure at a load of 61.7 kips after yielding of the longitudinal reinforcement was observed at both supports and at midspan. This ultimate load is identical to that of beam 2a. A view of beam 11 after failure is shown in Fig. 9. The axial tension of 10.2 kips (121 psi for the gross concrete section) did not reduce the flexural strength of the strapped test beam.

Beam 13—Tested at an average cylinder strength of 3300 psi, this beam was loaded to an axial load of 14.5 kips (173 psi tension) at a flexural load of 21.7 kips, and the flexural load was then increased until failure took place at 60.2 kips.

At an axial load of 10.7 kips plus the flexural load of 21.7 kips, a diagonal crack began to form under the third load from the north support. Developing upward from the bottom of the beam, this crack reached the top of the beam at an axial load of 14.5 kips. The crack was contained by one strap only. The flexural load was then increased. At the maximum load of 60.2 kips, which exceeds the calculated flexural capacity by 8 percent, extensive yielding had taken place in the longitudinal reinforcement at both supports and at midspan. As the maximum load was reached, other diagonal cracks formed

suddenly just north of the fourth load point in the unstrapped region, and diagonal cracks also appeared in the haunches.

This test indicated the type of strapped beam involved can exceed its calculated flexural strength by limit design, even at axial tension as high as 173 psi.

Beam 12—At an average cylinder strength of 3180 psi, this beam was also loaded axially at a flexural load of 21.7 kips. At an axial load of 16.0 kips, a steep crack formed just north of the third load from the north support.

At a flexural load of 21.7 kips, the calculated yield load by Eq. (5) is 16.0 kips and the separation load by Eq. (4) is 17.1 kips. At an axial load of 17.6 kips, the crack extended entirely through the beam between two straps, and the beam was in serious distress. The axial load was then maintained at 17.6 kips (210 psi) and the flexural load was increased until failure took place at 24.7 kips. The crack that caused failure is shown for beam 12 in Fig. 9.

In connection with the test results for beam 10, it was shown that separation or yielding of the tension reinforcement, rather than diagonal tension, could be the primary cause of failure. The fact that the strap reinforcement did not strengthen beam 12 significantly as compared to the unstrapped beam 10 strongly supports this possibility.

The test of beam 12 indicated clearly that if an axial tension as high as 210 psi is present, beams of the 1952 design fail by separation at flexural loads only slightly over the working load level. It is possible that the performance of beam 12 could have been improved by providing steel angles similar to those used in beam 2 across the failure crack.

Since the 1954 design has the same embedded reinforcement as the 1952 design plus top reinforcement and nominal stirrups throughout the beam, it is to be expected that similarly strapped beams of the 1954 design will develop their full flexural strength for axial tensions of at least the 173 psi sustained by beam 13.

Summary—The test results indicate that prototype beams of the 1952 and 1954 design, and strapped in a manner similar to test beams 11 to 13, should possess flexural and shearing strength equal to at least twice that needed to carry the full working load even if axial tensions as high as 173 psi (axial force per unit area of gross concrete section) should exist. As a matter of course, however, axial tensions of this magnitude due to temperature and moisture changes should be avoided by control in design and construction.

GENERAL DISCUSSION OF TEST RESULTS

Considerable discussion has been presented in connection with the presentation of test results for the various beams. It remains to consider deflection characteristics of the test beams and interaction concepts.

Deflection of test beams

Deflections were measured for all test beams by dial gages at midspan, at the quarter-points, and at the tie-down loads. Deflections for typical

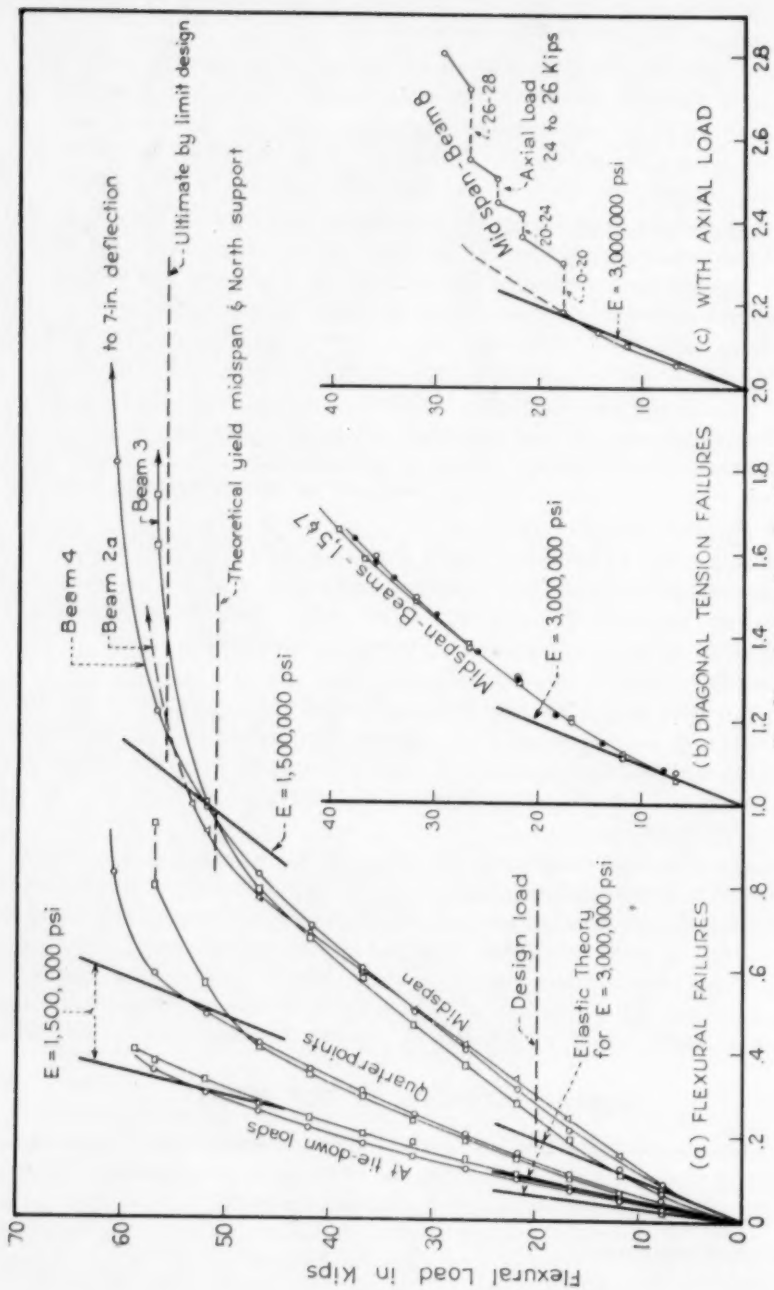


Fig. 13—Deflections of test beams

beams are shown as a function of flexural load in Fig. 13. Deflections at the quarter-points and at the tie-downs are plotted in the figure as averages of readings in the two halves of the beams.

The flexural failures, shown in Fig. 13a, were characterized by large midspan deflections after yielding had started in the longitudinal reinforcement over both supports and at midspan. Straight lines corresponding to deflections as computed by elastic theory (Fig. 2) are shown in the figure. At low loads, it is seen that the observed deflections follow closely the lines corresponding to $E = 3{,}000{,}000$ psi. At first yielding, however, the measured deflections correspond to a modulus of only 1,500,000 psi.

Deflection curves for all beams that failed in diagonal tension (Fig. 13b) followed the curves for flexural failures closely until the abrupt failure took place. The behavior of the beams with axial loading differed somewhat. When axial load was applied while a flexural load was being maintained, the beam deflection at midspan increased (Fig. 13c). This is to be expected because the longitudinal steel stress was increased by the axial load as shown by Eq. (7). As further flexural load was applied while the axial load was maintained, deflections continued to increase at the same slope as before the axial load was applied. This is shown by a dotted line in Fig. 13c, giving only the part of the deflection that is due to flexural load.

No abnormal characteristics of the test beam deflections were found. Similar deflection behavior for both flexural and diagonal tension failures has been observed and reported in the literature. The nature of the deflection curves for combined bending and axial load may be predicted by simple engineering mathematics.

Interaction concepts

Interaction diagrams are often convenient for study of combined loading phenomena. Such an interaction diagram is given in Fig. 14 for the contraflexure region of the test beams subject to combined flexural load and axial tension. To facilitate comparison with the failure of the prototype frames, both loads are expressed in terms of stress. The ordinate represents the shearing stress in the contraflexure region computed as $v = P_f/4bjd$. The abscissa represents the axial tension per unit area of gross concrete section. To permit comparisons, ultimate loads for beams that failed in the contraflexure region as well as for others that failed in flexure or in shear at other locations are plotted.

For the beams without axial load it may be said in summary that strapping (beams 2a and 3), or the 1955 redesign (beam 4), or design in accord with ACI 318-56 (beam 6) raised the ultimate shearing stress from about 150 psi to somewhat over 200 psi, a value at which flexure rather than diagonal tension governed ultimate strength.

Beams 9 and 10 failed at a low shearing stress of about 70 psi due to considerable axial tension. A line representing Eq. (5) is entered in Fig. 14, above and to the right of which yielding of the bottom reinforcement should

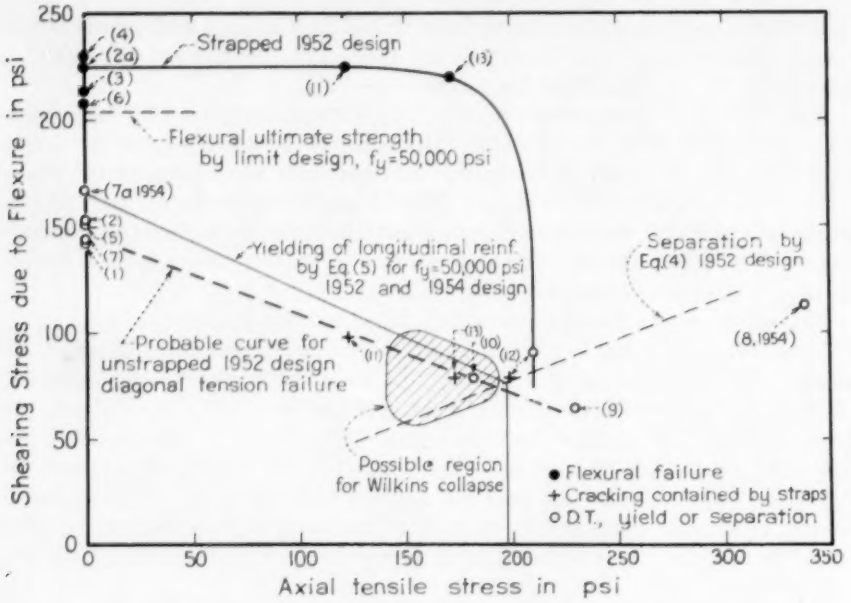

Shearing Stress due to Flexure in psi
Axial tensile stress in psi
Strapped 1952 design
Flexural ultimate strength by limit design, f_y=50,000 psi
Yielding of longitudinal reinf. by Eq.(5) for f_y=50,000 psi 1952 and 1954 design
Separation by Eq.(4) 1952 design
Probable curve for unstrapped 1952 design diagonal tension failure
Possible region for Wilkins collapse
(8,1954)
● Flexural failure
+ Cracking contained by straps
○ D.T., yield or separation

Fig. 14—Interaction diagram for contraflexure region

take place for the 1952 and 1954 designs at the weak section under the third load point from the supports. A second, broken line represents the condition of zero compression as given by Eq. (4). Below this line equilibrium at the third load point is theoretically possible for the 1952 design only if the concrete carries axial tension in the top of the beam.

It is seen that the reinforcement of beam 9 was yielding, and separation was possible. Beam 10 was on the verge of both yielding and separation, but it probably failed by diagonal tension. This is particularly probable because the strapped beam 13 cracked at a loading condition close to that of beam 10. Beam 13 subsequently sustained a large increase in flexural load, so that extensive yielding of its reinforcement must have taken place at the weak section. The strapping obviously prevented beam 13 from failing at a load combination close to that of beam 10. If a diagonal crack is defined as one that, contrary to flexural cracks, can be contained by vertical stirrups, beam 10 failed by diagonal tension. Another indication of the interaction line for diagonal tension is given by the strapped beam 11 which cracked at the load shown.

On the basis of these test data a tentative interaction line for diagonal tension failure of the 1952 design is drawn as a broken line in Fig. 14. Further tests are desirable to fix the position of this line. It seems possible that the diagonal tension interaction line may be influenced by its close proximity to the line for yielding of the bottom reinforcement at the weak section. This possibility may be investigated by further tests using high-strength reinforcement.

The failure load combination for beam 8 of the 1954 design is considerably beyond the yielding line in Fig. 14. The separation line shown in the figure is not applicable because of the top reinforcement provided. By Eq. (8), beam 8 could not separate. The interaction line between beam 8 and beam 7 should be established to investigate further the stability and strength of this type of design.

Consider then the failure of the Wilkins warehouse in terms of these interaction concepts. The reinforcing steel used had a high yield point, so the interaction line for yielding will be at a higher stress level than the line in Fig. 14—so high, in fact, as to virtually rule out yielding of the longitudinal steel as cause of failure. Calculated shearing stress for the Wilkins structure was about 75 psi at the failure crack due to dead load of girder and roof.

Shrinkage measurements on small specimens cut from the concrete specimen from Wilkins made available to the laboratory indicate a potential shrinkage of 0.05 percent from the saturated condition to equilibrum with 50 percent relative humidity. Temperature at the time the concrete set could have been as high as 130 to 140 deg for the 20 x 36-in. section involved. It is understood that the temperature inside the warehouses was later maintained at about 60 deg.

On this basis it is possible that an axial tension exceeding 140 psi was developed in the frame beams by volume change and the restraining action of the frame columns. A load combination region that possibly may represent the Wilkins collapse is tentatively shown in Fig. 14. In future analytical studies, the form and position of this region should be explored by rigorous structural analyses within the probable limits of shrinkage and temperature change for the Wilkins structure. It should be investigated to what extent differential shrinkage effects contributed to cause of failure.

CONCLUSIONS

Based on studies of test data for 13 laboratory beams representing 1/3-scale models of the prototype frame span that collapsed at the AMC warehouse of Wilkins Air Force Depot it can be concluded that:

(1) The location, inclination, and appearance of the crack that caused failure at Wilkins was closely reproduced in the laboratory (beam 10, Fig. 6).

(2) The revised frame design prepared by the Office of the Chief of Engineers and their consultants in October, 1955, may be expected beyond all reasonable doubt to be adequate for new construction.

(3) Steel strapping applied as external web reinforcement with a prestress exceeding 30,000 psi to frames that are in distress should be an adequate remedial measure. If such strapping is designed by the equation $A_s = Vs/f_s jd$ for not less than the total working load shear, V, at a strap stress, f_s, of 50,000 to 60,000 psi including the prestress, and if strapping so designed is carried well beyond the extreme position of the point of contraflexure, then the strapped frames should possess ample reserve structural strength.

(4) It seems assured that, in addition to shearing and flexural stress due to dead load, tensile stress in the longitudinal frame direction was developed by shrinkage and temperature change. The laboratory test data indicate that failure of the Wilkins structure can be explained in terms of a combined loading.

(5) Adequate safety in structures of the Wilkins type may be assured in future designs by the following three measures: (a) sufficient web reinforcement, (b) sufficient extension of both positive and negative longitudinal reinforcement well beyond the extreme positions of the point of contraflexure, and (c) effective expansion joints.

(6) The fundamental mechanism by which the collapse at Wilkins took place is, as yet, not entirely clear. There are strong reasons to believe, however, that the failure mechanism was: (a) diagonal tension under combined flexure and axial tension, or (b) separation under high axial tension due to insufficient extension of top reinforcement beyond the computed contraflexure region, or a combination of (a) and (b). Effects of differential shrinkage were probably also present.

(7) Further experimental and analytical investigation of a more fundamental nature should follow the study of a specific practical case reported herein.

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Title No. 53-36

Symposium on AMC warehouse failure—remedial measures

Application of Steel Strap Reinforcement to Girders of Rigid Frames, Special AMC Warehouses*

By REINHART R. LUNOE and GEORGE A. WILLIS†

SYNOPSIS

The test applications reported were made to develop a procedure for installing the steel strap reinforcement on full-scale girders in the field, and to work out solutions for any impractical features of the design specifications. Test applications were made on one rigid frame on which the roof deck had been placed and on one rigid frame without the roof deck. Buttering the inside of the corner protection angles with grout prior to clamping the angles in place was found to be the most practical method of insuring uniform bearing between the angles and the concrete girders. A standard model stretcher calibrated by a torque indicator attached to the operating handle is satisfactory for tensioning the straps to a reasonably accurate predetermined stress. Tapping the straps at the corners of the girder both during the tensioning operation and after sealing was necessary to develop higher stresses and to produce more nearly equal stresses in the straps on all sides of the girder.

INTRODUCTION

As a result of the widespread cracking of girders which had occurred in several completed special AMC warehouses, the Office of the Chief of Engineers engaged Ammann and Whitney, consulting engineers, to investigate the possible cause of the cracking‡ and to develop remedial measures to restore the design strength of the girders. In addition, three nationally recognized consultants in the field of reinforced concrete design, N. M. Newmark, C. P. Siess, and R. J. Hansen,§ were engaged to review the studies and designs prepared by Ammann and Whitney.

To increase the deficient strength in diagonal tension of the girders, a girder reinforcement consisting of a series of stirrups, or straps, to be placed around the girders and then prestressed and sealed, was proposed by the Chief of Engineers and the design was developed by Ammann and Whitney. The effectiveness of this method of girder reinforcement was confirmed by laboratory tests on 1/3-scale models in the Research and Development Laboratories of the Portland Cement Association.**

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‡Anderson, Boyd G., "Rigid Frame Failures," ACI JOURNAL, Jan. 1957, *Proc.* V. 53, pp. 625-636.

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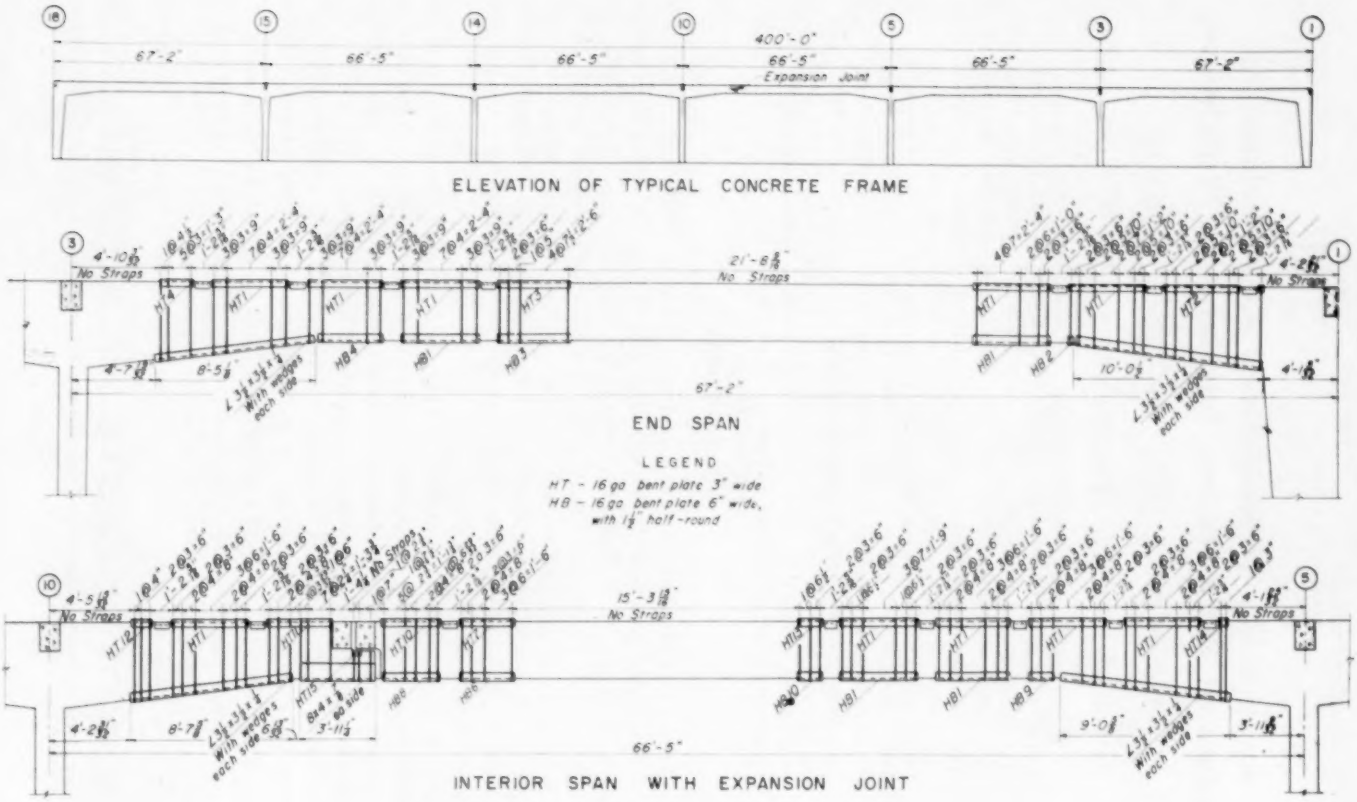

Fig. 1—Typical spacing of steel strap reinforcement

In November, 1955, the Office of the Chief of Engineers directed the Savannah District to conduct prototype test installations of the successfully model-tested exterior prestressed steel strap reinforcement. Test installations were made on the girders of two rigid frames in special AMC warehouse No. 4 at Robins Air Force Base, Ga. This paper describes briefly the material, equipment and methods, and the result of these installations.

Preliminary tests were conducted in December, 1955, to develop satisfactory methods for installing corner protection and straps, and to test and calibrate different types of stretchers for prestressing the straps. Final installation of straps on the 12 girders of the two frames was started December 28, 1955, and completed January 24, 1956. Corner angles and straps were placed and spaced essentially as shown on Fig. 1. On the basis of additional model studies by the Portland Cement Association, subsequent to the prototype experiments, it was deemed desirable to extend the corner angles and straps to the column faces instead of being stopped approximately 4 ft from the columns as shown on Fig. 1.

MATERIALS AND EQUIPMENT

The 2 x 0.05-in. steel straps, stretchers, seals, and sealers used in the test application are all items standard with the strapping and banding industry. A brief description of the materials and equipment used, along with applicable specifications and sources, is given in Table 1. Steel corner protection was

TABLE 1—LIST OF MATERIALS AND EQUIPMENT

Item	Specification	Furnished by
Galvanized steel straps	Federal Specification QQ-S-781b, dated Aug. 3, 1954. Class B, zinc-coated. Grade I—heavy. Breaking strength not less than 100,000 psi; yield stress not less than 70,000 psi. Size: 2 x 0.05 in.	Gerrard Steel Strapping Co.
Seals	Lithograph seal No. 208	Signode Steel Strapping Co.
Stretcher	Signode Model W-2	Signode Steel Strapping Co.
Cutter	Gerrard strap cutter No. PS 215	Gerrard Steel Strapping Co.
Sealer	Signode manual sealer No. 250	Signode Steel Strapping Co.
Torque indicator	Proto-Tool torque indicator Model 6016 (150 ft-lb)	Plumb Tool Co.
Lubricant	Liqui-Moly regular molybdenum disulphide powder- Specification MIL-L-7865b	Lubricants Division, Lockray Co.
Bitumastic paint	MIL-C-18480	—
Corner angles	ASTM A 7-53T	—
16-gage plates	ASTM A 7-53T	—
Wedges	AISI C1012 (Forged)	—
1½-in. half-round	AISI C1012	—

placed on the girders to provide a smooth contact surface for the straps and to distribute the load under the straps. Bent plates, formed from 16-gage sheet steel, were used on the top corners of the girders and on the bottom corners on the prismatic portion of the girders. Standard rolled angles were used on the bottom corners on the haunches. To provide space between the tensioned strap and the bottom of the girder for the base plate of the stretcher, 1½-in. diameter half-rounds were tack welded by resistance welding to the bottom corner protection plates. Wedges were welded to the corner protection angles providing a bearing surface for the straps parallel to the top of the girder to assure uniform stress distribution across the width of the straps at the haunches. The wedges were spaced to fit the strap spacing and were placed with the edge of each wedge projecting 1/16 in. beyond the heel of the angle.

STRAPPING APPLICATION

Corner protection

The concrete girders to be strapped had a ¾-in. chamfer on the bottom corners but no chamfer at the top. To eliminate the sharp right angle bend in the strap at the top of the girder the corners were ground to a ¼ in. bevel. Corner protection plates were bent in the shop to conform to the chamfer and the bevel, and were furnished in 3-in. lengths, in lieu of the longer lengths indicated in Fig. 1.

To provide solid bearing between the corner protection angles at the haunches and the girder, the space in the chamfer and any other deformation were filled with neat cement grout. A satisfactory method of pressure grouting was developed by providing ¾-in. grout holes and using a heavy-duty calking gun. Grout holes were spaced about 3 ft on centers with lower point of holes flush with inner face of the horizontal leg of the angle. The use of pipe nipples was found unnecessary as sufficient grout pressure was maintained by pressing the gun snout firmly against the grout hole. However, the most satisfactory and economical method of obtaining solid bearing was to prepare the grout to a buttering consistency and spread it on the inside of the angle prior to clamping the angle in place.

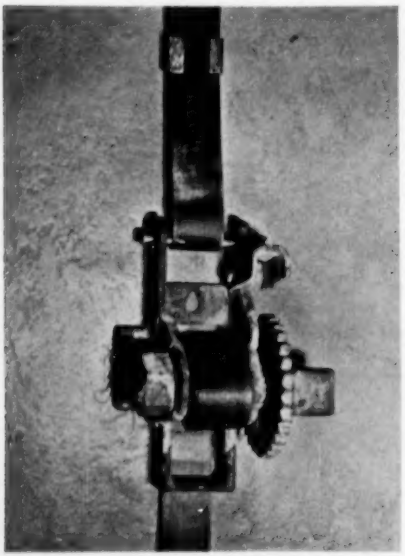

Fig. 2—Bull-winch column clamp for holding corner angles temporarily in place

The angles were positioned and held temporarily in place by use of a modified "bull-winch" column clamp and 1¼ x 0.035-in. ungalvanized steel strap. This device was readily removable for continuous re-use. Only the main ratchet attachment of the clamp was used with a 5/16-in. pin provided at the dogging end of the ratchet; 1/8-in. holes were drilled at both ends of the pin for insertion of removable cotter pins, and the dead end of the strap was looped and sealed and placed on the 5/16-in. pin (Fig. 2). For ease of application the strap was installed with the ratchet on the side of girder. Temporary straps were installed about 4 ft on centers, with two straps minimum for one angle.

Calibration of stretcher

Tensioning of the straps was done with a Signode Model W-2 stretcher. This stretcher is equipped with reduction gears and required a relatively small force to produce a high stress in the straps. In order that a reasonably accurate predetermined stress could be induced in the straps, the stretcher was calibrated by attaching a torque indicator to the stretcher handle. The torque indicator, of 150 ft-lb capacity, was attached by cutting off one handle of the stretcher about 6 in. from the centerline of the shaft and welding to the handle a 3/4-in. diameter standard pipe 12 in. long. A nut fitting the indicator was then welded to the pipe 12 in. from the centerline of the stretcher shaft. The stretcher was calibrated by applying a force until a predetermined torque set on the indicator had been reached, and then, by use of Berry strain gages and SR-4 electric strain gages, measuring the stress produced in the straps.

With this assembly of stretcher and torque indicator, it was found that the average stress induced in the strap on the side of the girder near the feed side of the stretcher was approximately 900 times the torque reading. Table 2 shows the average stress values obtained for various torque readings. Depending on carefulness of tapping the strap during stretching operations and amount of friction around the girder corners, the stress in the strap on the opposite side of the girder near the dogging side of the stretcher will generally be from 5 to 20 percent less, and in the strap over top of girder from 10 to 30 percent less than the stresses indicated in Table 2. After sealing and tapping, stresses in the strap on the sides of the girder will generally be 15 to 35 percent (with an average of 20 percent) below the stresses recorded before sealing.

TABLE 2—STRETCHER CALIBRATION DATA

Torque indicator reading, ft-lb	Stress in strap, psi side of girder near feed side of stretcher
40	1,080 × torque
45	980 × torque
50	940 × torque
55	910 × torque
60	880 × torque

Tensioning operation

Steel straps were installed following the procedure outlined below.

1. All contact surfaces on angles, wedges, half-rounds, and corner plates

were cleaned carefully with power driven wire brushes or grinding wheels to provide a smooth contact surface free of all scale and projections.

2. Installation of straps on each girder was started at one end of a group of straps between roof slab bearing plates and worked toward the other end so that room was always maintained for operation and removal of the stretcher. The temporary column clamps holding the corner protection angles on the haunches were left in place until installation of the permanent straps made their removal necessary.

3. Lubricant was applied to contact surfaces of wedges, corner plates, and half-rounds just prior to installation of straps.

4. The straps were tensioned in accordance with the following procedure:

(a) Stretcher was placed on the bottom of the girder to obtain nearly equal stresses in the strap on the sides of the girder.

(b) Strap was placed in the drum slot of the stretcher through the full length of the slot without projecting through and with sufficient slack left in the strap to provide a minimum of $1\frac{1}{2}$ windings on the drum at a maximum torque of 10 ft-lb on the indicator. Fig. 3 shows the beginning of the tensioning operation.

(c) Torque indicator was set at 35 ft-lb and strap tightened slowly, the strap being tapped at the corners of the girder during the tightening operation.

Fig. 3—Beginning tensioning operation—note slack in strap

Fig. 4—Tensioning operation —force being applied on torque indicator

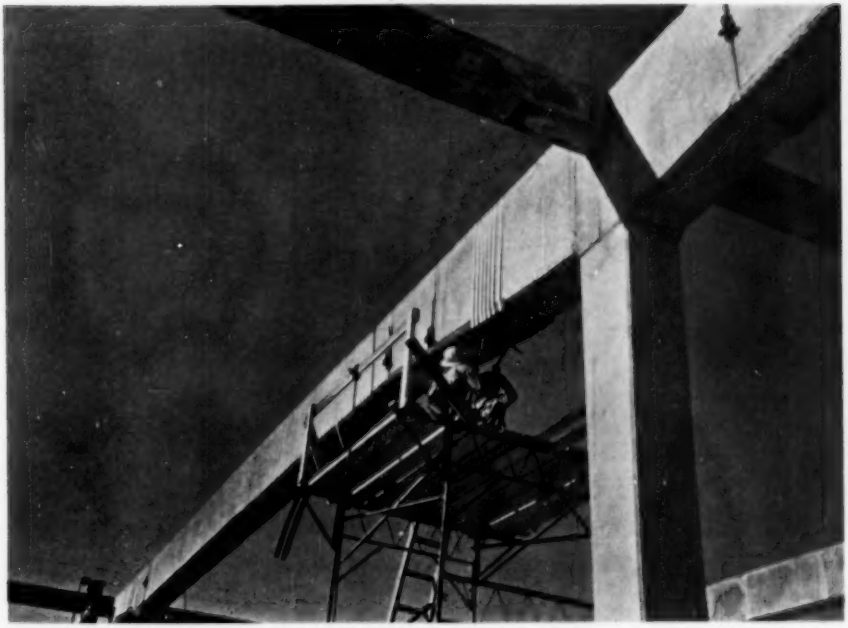

Fig. 5—Straps being installed on girder 15-18. Note column clamps and straps used for holding angles temporarily in place

(d) Strap was tightened from the 35 ft-lb torque to a final torque of 65 ft-lb in increments of 5 ft-lb, and tapped at the corners of the girders at the end of each increment. Fig. 4 shows tensioning force being applied through the torque indicator.

(e) Strap was sealed using two seals with two crimps in each seal and the corners again tapped to distribute stress in strap around the girder.

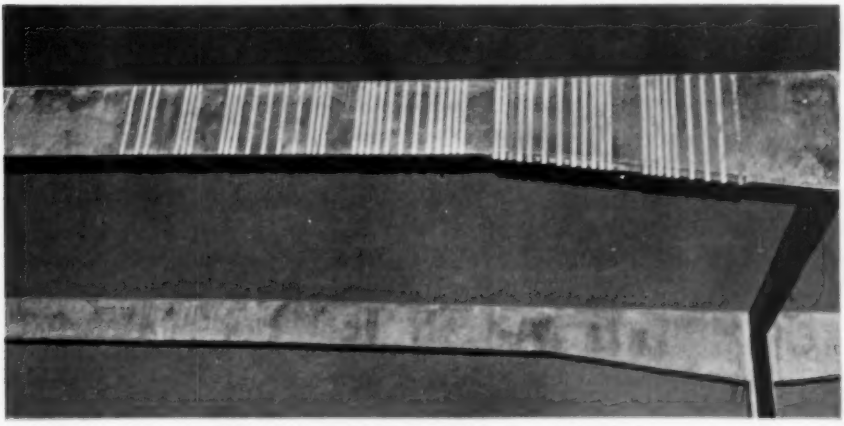

Fig. 6—Strapping completed on girder 3-5

Fig. 7—Strapping completed at expansion joint

(f) The ends of strap and the seals were painted with bitumastic paint immediately after crimping the seals.

Fig. 5 is a general view of straps being installed on the girders of a rigid frame without roof decking. Fig. 6 and 7, respectively, show completed sections of strapping on a girder without roof deck and on a girder with roof deck in place.

Measurement of stresses

To measure the actual stress in the straps after sealing and tapping, approximately one of every 20 straps was selected at random and tested with a Berry strain gage by taking readings on each side of the girder, and then cutting the strap to obtain the zero stress reading. The maximum stress measured was 55,100 psi and the minimum stress measured was 28,600 psi. The average stress in both sides of all the straps tested was 42,700 psi.

Special tests were made on three straps to determine the stresses in the straps at the bottom and sides of the girder during the tensioning operations and after the seals had been placed. The special tests indicated that stressing the straps beyond their yield point on the bottom of the girder during tensioning will not produce materially higher stresses in the straps on the sides of the girders after sealing and tapping than when the bottom portion of the straps is stressed to about 60,000 psi. Results of the special tests are given in Table 3. It will be noted from the table that the bottom portions of straps A and B were stressed beyond the yield point before seals were applied. It will also be noted that the resulting stresses after sealing in the side portions of straps

TABLE 3—STRESSES IN TEST STRAPS DURING AND AFTER TENSIONING

Test No.	Torque, ft-lb	Stress in 1000 psi		
		Bottom	Feed side	Dog side
A	30	30.4	27.9	26.6
	40	46.6	37.0	35.2
	50	63.8	48.2	46.4
	60	72.0+	62.4	54.8
	After sealing		34.5	33.5*
B	40	42.6	40.2	37.6
	50	51.6	52.5	47.0
	60	72.0+	67.5	64.7
	After sealing		55.3	55.3†
C	30	31.3	29.2	27.0
	40	46.0	36.4	36.4
	50	54.8	38.3	37.1
	55	57.9	43.2	39.0
	After sealing	33.0	33.2	30.0*

*Strap tapped after sealing.
†Strap not tapped after sealing.

A and C are practically the same. The corners of straps A and C were tapped after sealing, while the corners of strap B were not tapped after sealing.

MAN-HOURS AND COST OF STRAP INSTALLATION

During the strap installation on the 12 girders of the two rigid frames described in this paper, an average of 37 straps was installed in an 8-hr working day by a crew consisting of two carpenters and one laborer, resulting in a total installation time of 24 hr, or an average of 0.65 man-hr for each strap.

In July, 1956, a contract was let for installation of 22,500 steel straps on the girders of 36 rigid frames of three AMC warehouses under construction at Robins Air Force Base. During the initial period of the strap installation, a three-man crew installed an average of 46 straps per 8-hr day, or an average installation time of 0.52 man-hr per strap. During the final period of the contract, a three-man crew installed an average of about 65 straps per 8-hr day, or an average installation time per strap of 0.37 man-hr. The average man-hours required refer only to installation of the straps; the corner angles were already grouted in place prior to strap installation.

The contract for the strap installation consisted of furnishing all material, equipment, and labor required for installation of the corner angles, corner protection plates, and straps, and included replacing one out of about every 20 straps installed, which was cut and removed for testing purposes. The contract price for installing the 22,500 straps was $150,000, or an average installation cost of $6.70 per strap. The three warehouses were unoccupied, and the roof consisted of long-span channel slabs between the rigid frames, providing reasonable space for inserting the straps between the top of the girders and the roof slabs.

CONCLUSIONS

1. Steel strap reinforcement to strengthen girders deficient in diagonal tension strength can be applied practicably in the field.

2. The surfaces of angles, wedges, and half-rounds in contact with the straps should be as smooth as practicable and well lubricated to obtain the maximum tension in the straps after sealing. A surface finish of 125 micro-in. or less is being specified for future strapping applications.

3. Better production can be obtained if the corner protection plates are furnished in as great a length as practicable, eliminating the labor of holding the short individual plates in position while the straps are being installed.

4. Tapping the straps at the corners of the girder during tensioning reduced the resistance to slippage due to the bend in the straps and due to friction between the straps and the corner plates and angles, resulting in higher stresses being induced in the straps on the sides of the girder.

5. Since the effectiveness of the strap installation is dependent upon maintaining a given minimum stress in the straps on the sides of the girder, tests should be made on representative straps at the time of application to determine if the desired stresses are obtained. During the tensioning operation the free end of the strap extending beyond the dog on the stretcher is not stressed. For this reason, after the strap is sealed and the stretcher removed, the stress in the strap on the bottom of the girder is considerably less than the stress induced during the tensioning operation, and less than the stress induced in the strap on the sides. The straps should be tapped at the corners of the girder immediately after sealing, to distribute the stress, so that stress measurements taken at the time of application will represent the stresses which may be expected over a long time period.

6. It is practicable to set 30,000 psi as a minimum requirement for the stress in any portion of a strap after sealing.

7. The fire rating which was provided in the warehouse framing as originally designed can be restored after the installation of the steel straps by covering the straps with fireproofing plaster.

ACKNOWLEDGMENTS

The installation of the steel strap reinforcement was done by Thompson and Street Co., general contractor for construction of the warehouses, under the direction of the District Engineer, Savannah District, Corps of Engineers. Calibration of the stretcher and measurements of stresses in the installed straps were done by the South Atlantic Division Laboratory, Marietta, Ga.

Discussion of this paper should reach ACI headquarters in triplicate by May 1, 1957, for publication in the Part 2, December 1957 JOURNAL.

Title No. 53-37

Recent Investigations of the System Lime-Alumina-Calcium Sulfate-Water and its Importance in Building Research Problems*

By WILHELM EITEL†

SYNOPSIS

The general character of the complex calcium aluminate sulfate hydrates in the quaternary system is outlined on the basis of the previous publications of F. E. Jones and of revised determinations of the concentrations of the equilibrium solutions. New projection methods for the quaternary equilibria are given which are particularly obvious for solutions of very low concentrations in lime, calcium sulfate, and alumina hydrate. The importance of unstable crystallizations is emphasized, especially the formation of crystalline solutions of tetracalcium aluminate hydrate and $3CaO \cdot Al_2O_3 \cdot CaSO_4 \cdot 12H_2O$. The use of the revised equilibrium diagram for the explanation of the reactions of sulfate solutions with pozzolans containing hydraulic binders is particularly illustrated. The role of activated alumina in zeolitic weathering minerals of pozzolanic and related rock types is emphasized for the correct interpretation of such reactions.

INTRODUCTION

In the evolution of modern civil engineering, no destructive phenomenon has perhaps found such great interest from the viewpoint of applied scientific research as the deterioration of portland cement concrete in sulfate-containing waters (including sea water). A wealth of publications in this field have been written, and the mysteries of a hidden reaction in the interior of the concrete body have attracted scientific investigators for several decades. Discovery of an apparent "cement bacillus" was a real sensation,[1] but not before 1917[2] was this renowned compound identified in deteriorated concrete. This "cement bacillus" has the same composition as the natural mineral ettringite[3] which is also identical with the synthetic product $3CaO \cdot Al_2O_3 \cdot 3CaSO_4 \cdot 32H_2O$, first described by Candlot[4] with its characteristic fine needles and radial spherulite groups.

We owe a very careful investigation[5] on the synthesis of this interesting material to the indefatigable research staff of the Portland Cement Association Fellowship at the National Bureau of Standards which, in addition, gave full evidence that another compound exists of the composition $3CaO \cdot Al_2O_3 \cdot CaSO_4 \cdot 12H_2O$. This latter double salt crystallizes in characteristic

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hexagonal tables (often aggregated to star-shaped spherulites) and is isomorphous with the "aluminate hydrate" $4CaO \cdot Al_2O_3 \cdot 13H_2O$ (known as the mineral hydrocalumite).[6] We may write the formula of this latter compound as $3CaO \cdot Al_2O_3 \cdot Ca(OH)_2 \cdot 12H_2O$ to illustrate the analogous chemical constitution. Both compounds form a series of crystalline solutions;* therefore, the ratio $CaSO_4 \cdot Al_2O_3$ may vary between the limits of 1:1 to zero. In the following, we shall call the compound $3CaO \cdot Al_2O_3 \cdot CaSO_4 \cdot 12H_2O$ the "low-sulfate" aluminate-sulfate double salt,[7] to distinguish it from that with $CaSO_4:Al_2O_3 = 3:1$ (ettringite), which logically is the "high-sulfate" double salt.

PREVIOUS INVESTIGATIONS OF THE PHASE EQUILIBRIA IN THE SYSTEM (Ca^{2+}, Al^{3+}) (SO_4^{2-}, OH^-)–H_2O

In the spirit of the classical investigations of van't Hoff and his school on the equilibria of the formation and decomposition of double salts from aqueous solutions,[8] the physical-chemical discussion of compounds like ettringite and of the type of the other compounds mentioned above must be based on accurate determinations of the solubility and the crystallization in the fundamental salt-water systems. In the present problem of the formation of calcium aluminate sulfates,† the reciprocal equilibria of reactions in the complex system $Ca(OH)_2$-$Al(OH)_3$-$CaSO_4$-$Al_2(SO_4)_3$-H_2O are concerned, including the ionic reactions in aqueous solutions between the cations Ca^{2+} and Al^{3+}, and the anions SO_4^{2-} and OH^-. The isothermal crystallization and dissolution equilibria were first investigated in a most remarkable work by Jones,[9] which Steinour[10] extensively evaluated for use by cement chemists in a special monograph on aqueous cementitious systems, and which was extended to solutions containing alkalies by Kalousek[11] and Jones.[7]

For a fundamental understanding of the character of complex reciprocal systems, we want first to illustrate very briefly the essential reactions represented by graphic projection of the type used by Jones. We generally define as "reciprocal salt pairs" those aqueous solution systems whose components are formed by two cations (M' and M''), and two anions (A' and A'') with equal sums of cations and anions, *i.e.*, in the present case of Ca^{2+} and Al^{3+} in relation with SO_4^{2-} and OH^-. The "double conversion" of the simple compounds $M'A' + M''A''$ in saturated solutions into $M'A'' + M''A'$ means, for a given constant temperature, the instability of the mixture of the initial salts on the left side, and the stability of those on the right side of the equation in the crystallization end products. In the graphic projection, Fig. 1, the molar proportions of the components in aqueous solutions saturated with respect to the crystalline phases are plotted.

Usually, compositions in reciprocal systems are expressed by the rectangular

*The term "crystalline solutions" is used throughout the present text instead of the more common "solid solutions." The reasons why preference must be given to "crystalline solutions" for a correct translation of the German definition of "Mischkristalle" are discussed by W. Eitel in *Silicate Melt Equilibria*, Rutgers University Press, 1951, par. 44f; and *Physical Chemistry of the Silicates*, University of Chicago Press, 1954, pp. 69-85.

†The term "calcium aluminate sulfate" is used in preference to the widespread but misleading shorter term "sulfoaluminate." A true sulfoaluminate is, for example, $CaS \cdot Al_2S_3$. "Calcium aluminate sulfate" also conforms to the usage of the American Chemical Society in its *Index to Chemical Abstracts*, V. 47, 1953, "Subject Index," p. 326.

coordinates x for the cations M'', $(100 - x)$ for the cations M', and y for the anions A'', $(100 - y)$ for the anions A', plotted in molecular percentage coordinates, respectively. The areas into which Fig. 2 is divided contain all the points representing saturated solutions from which the corresponding component salts crystallize. The boundary curves represent solutions saturated with two salts, the "invariant" points E, F are single solutions saturated with three salts. The arrows on the boundary curves indicate the progressive crystallization during isothermal evaporation of the solution (*i.e.*, at constant temperature, *e.g.* 25 C). In Fig. 2, the combination of $M'A'' + M''A'$ behaves as a binary partial system, *i.e.*, no other crystal phase appears than these salts during isothermal evaporation. The invariant ternary equilibria in the points E and F in Fig. 2 have a "congruent" character,[12] *i.e.*, the saturated solutions E and F are projected inside the composition triangle of the crystalline phases $M'A' + M''A' + M'A''$ (for E), and $M'A'' + M''A'' + M''A'$ (for F). Fig. 3, on the other hand, illustrates another type of reciprocal relationship in which the combination $M'A'' + M''A'$ is only partially binary because, in the middle region, neither $M'A''$ nor $M''A'$ crystallizes, but $M''A''$ does. This latter crystal phase, however, reacts with the solution phase when, by isothermal evaporation, the point F is reached. This invariant point F represents the incongruent reaction of the saturated solution of the composition F, with the primary crystals of the salt $M''A'':M''A'' + F \rightarrow M'A'' + M''A'$.

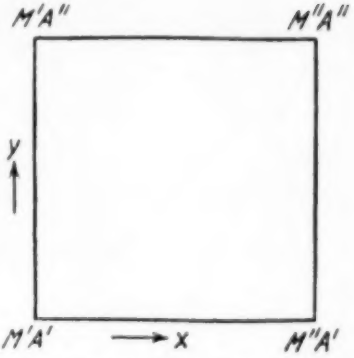

Fig. 1 Graphic representation of a reciprocal salt pair

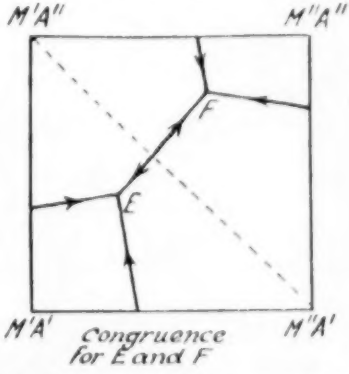

Fig. 2 Congruency of the compositions E and F in a reciprocal salt system

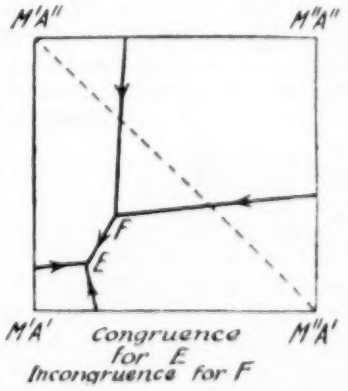

Fig. 3 Congruency for composition E, incongruency for composition F in a reciprocal salt system

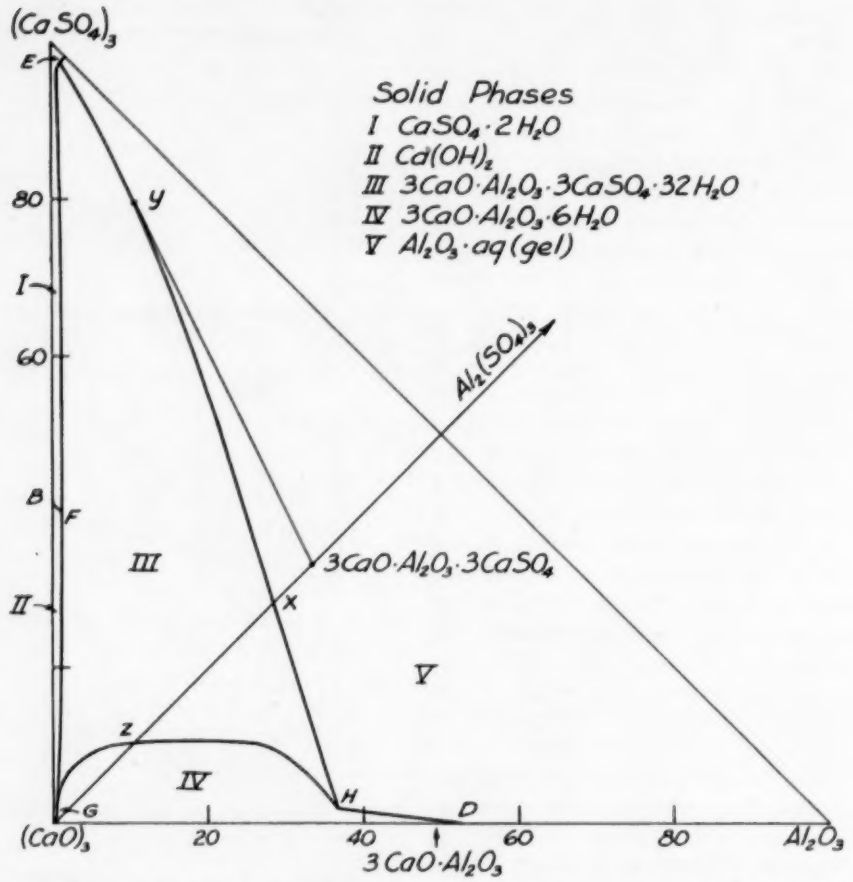

Fig. 4—System CaO-Al_2O_3-$CaSO_4$-H_2O, for the constant temperature of 25 C (Jones). Alumina hydrogel is the metastable precipitate in the field of primary (stable) crystallization of hydrargillite. (Diagram slightly distorted at the left side to show the fields of primary crystallization more distinctly)

Point F is projected outside the composition triangle of the crystalline phases $M'A'' + M''A'' + M''A'$.

In the special case of the calcium aluminate sulfate systems, Jones chose as the components (see Fig. 4) the anhydrous molecules Al_2O_3 for $M''A'$, $(CaO)_3$ for $M'A'$, $(CaSO_4)_3$ for $M'A''$, and $Al_2(SO_4)_3$ for $M''A''$. The multiple formulae $(CaO)_3$ and $(CaSO_4)_3$ are used here only to equalize the number of molecules on both sides of the equation defining the reciprocal relationships, namely $(CaO)_3 + Al_2(SO_4)_3 \rightleftharpoons Al_2O_3 + (CaSO_4)_3$. The stable diagonal section is that from Al_2O_3 to $(CaSO_4)_3$; ettringite, the point representing which is plotted on the unstable diagonal $(CaO)_3$–$Al_2(SO_4)_3$, is immediately characterized as an incongruent double salt which is hydrolyzed by pure

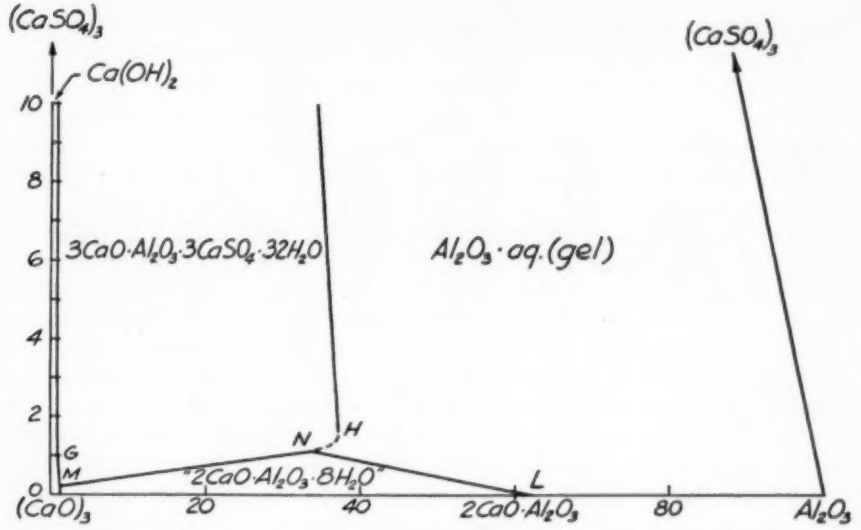

Fig. 5—Metastable crystallization of $2CaO \cdot Al_2O_3 \cdot 8H_2O$, as initially established by Jones

water with the precipitation of an alumina hydrogel. In other words, the ettringite point is placed in the wide field of primary precipitation of alumina hydrate and outside of its own field of primary crystallization from saturated solutions (Fig. 4).

Aqueous systems with highly charged ions typically may show very obstinate supersaturation nonequilibria. Therefore, the precipitation of the free hydroxides, which in true equilibria should form crystallized phases, is very sluggish and the products often appear as colloidal hydrogels. The stable crystal phase of alumina hydrate, $Al_2O_3 \cdot 3H_2O$ = hydrargillite, therefore, often does not appear as the primary precipitation product, but instead as a flocculant hydrogel. Even calcium hydroxide is much inclined to form unstable supersaturated solutions and colloidal suspensions characteristic of a slowly growing cryptocrystalline material. Such metastable phenomena, which also mean very stubborn nonequilibria in the fields of the primary crystallization of the calcium aluminate hydrates, $3CaO \cdot Al_2O_3 \cdot 6H_2O$, and $2CaO \cdot Al_2O_3 \cdot 8H_2O$, were overcome by Jones by an intense stirring of the reacting solutions and a "seeding" with crystal nuclei of the compounds concerned.

Jones' results are a remarkable first contribution to the solution of the ettringite problems in showing this compound as the only stable double salt. The boundary curves surrounding its field of primary crystallization are, however, different in the stable reciprocal isotherm diagram, in its first form (Fig. 4), and the metastable, experimentally more easily established isotherm (for a constant temperature of 25 C, Fig. 5). The intersection

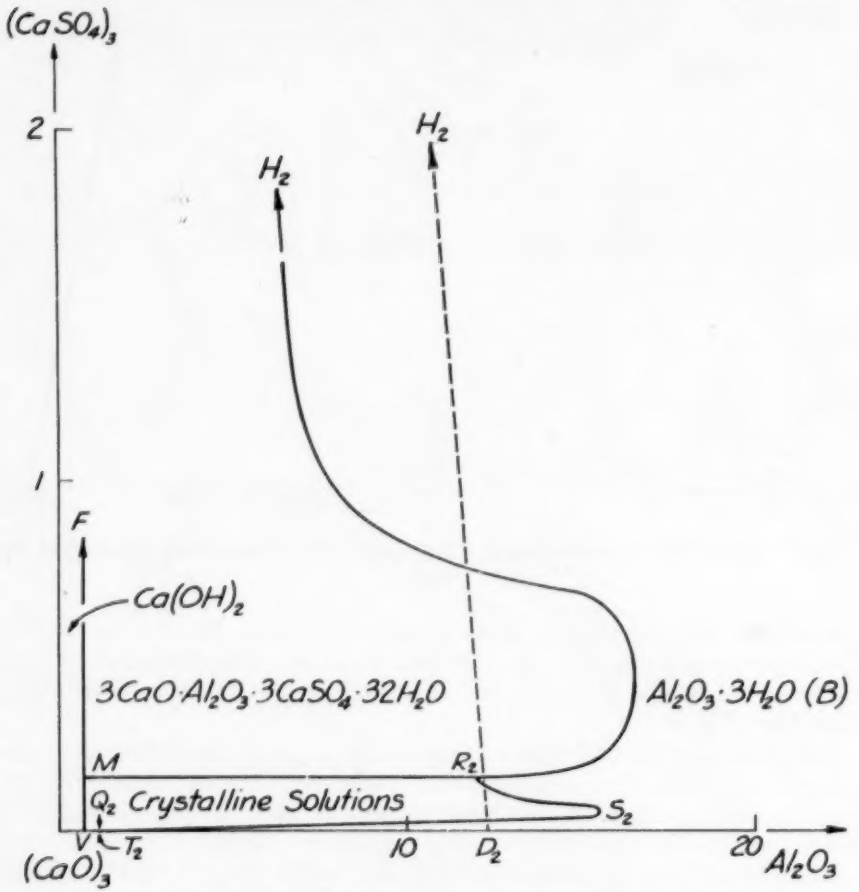

Fig. 6—Portions of the metastable crystallization diagram for the system CaO-Al_2O_3-$CaSO_4$-H_2O, at 25 C, with crystalline solutions of $3CaO \cdot Al_2O_3 \cdot CaSO_4 \cdot 12H_2O$ with $3CaO \cdot Al_2O_3 \cdot Ca(OH)_2 \cdot 12H_2O$ (later form as given by Jones)

point X of the system with alumina hydrogel (Fig. 4) shows the composition of this solution beyond which, with increasing calcium hydroxide concentration, the ettringite becomes congruently soluble, *i.e.*, it no longer is decomposed by the aqueous solution. Similarly, point Y shows the critical concentration of calcium sulfate beyond which the ettringite is also not decomposed, even if the free lime hydrate is decreasing. The intersection point Z on the boundary curve G-H characterizes the highest concentration of calcium hydroxide beyond which the ettringite is changed into $3CaO \cdot Al_2O_3 \cdot 6H_2O$ (hexahydrate). But this reaction is extremely sluggish. The metastable system (Fig. 5) does not show any direct transition of ettringite to an aluminate hydrate with increasing concentration of lime hydrate. The strong

supersaturation of calcium hydroxide, which is so important for the setting of portland cement-water mixes, on the other hand may be favorable for the unstable crystallization of the low-sulfate double salt if, at the same time, the concentration of $CaSO_4$ is low. Even the formation of the crystalline solutions mentioned above is possible. Jones, in his first investigations,[9] found neither the low-sulfate compound nor $4CaO \cdot Al_2O_3 \cdot 13H_2O$ as an equilibrium product. Alumina hydrates are in the stable equilibrium as crystalline $Al(OH)_3$ (in the α modification as bayerite, in the γ-modification as hydrargillite), but usually appear as hydrogel precipitates. The phenomena are even more complicated by the metastable crystallization of the originally assumed aluminate hydrate $2CaO \cdot Al_2O_3 \cdot 8H_2O$, which was shown to coexist with ettringite along the boundary curve M-N in Fig. 5. Later, however, Jones revised his results[7] and showed that, for very low concentrations of calcium sulfate, the metastable coexistence of ettringite with hydrargillite is possible along the boundary curve H_2-R_2 (Fig. 6), and that the crystalline solutions of the low-sulfate double salt with $4CaO \cdot Al_2O_3 \cdot 13H_2O$ (described as characteristic hexagonal plates with variable refractive indices) replace the needles of the high-sulfate compound (ettringite), with at the same time precipitation of hydrargillite along the peculiarly shaped boundary curve R_2-S_2-T_2. The boundary curve MR_2 practically coincides with M-N in Fig. 5.

NEW ASPECTS OF THE SYSTEM (Ca^{2+}, Al^{3+}) (SO_4^{2-}, OH^-)–H_2O

An extensive recent revision of the equilibria in the system $Ca(OH)_2$-$Al(OH)_3$-$CaSO_4$-$Al_2(SO_4)_3$-H_2O in the publications of J. D'Ans and H. Eick[13] is based chiefly on an improved orthogonal projection of the invariant points, boundary curves, and crystallization surfaces in a three-sided pyramid standing upright with H_2O at the apex (Fig. 7). The partial systems CaO-$CaSO_4$-H_2O; CaO-Al_2O_3-H_2O; and $CaSO_4$-Al_2O_3-H_2O[14] are plotted as the side faces of the pyramid. The solubilities of the investigated complexes are directly plotted in mg per l of solution; they were determined by D'Ans and Eick for the constant temperature of 20 C (Jones chose 25 C). Immediately, this type of projection makes evident the large boundary surface for the *stable* crystallization of ettringite (GFE_2H_2) which lies close to the plane CaO-$CaSO_4$-H_2O. Adjacent to this surface are the surfaces for the crystallization of gypsum (E_2j_2bcF), for $Ca(OH)_2$ (cFGtw), $3CaO \cdot Al_2O_3 \cdot 6H_2O$ (Gts_2H_2), and hydrargillite ($E_2H_2s_2a_2j_2$). All these surfaces enclose the space containing the points representing unsaturated solutions. The boundary curves for two coexisting crystalline phases are c-F for gypsum + $Ca(OH)_2$, t-G for the hexahydrate + $Ca(OH)_2$, s_2-H_2 for hexahydrate + hydrargillite, and j_2E_2 for gypsum + hydrargillite. Three crystalline phases coexist in the points F for $Ca(OH)_2$ + gypsum + ettringite, G for $Ca(OH)_2$ + hexahydrate + ettringite, H_2 for hydrargillite + hexahydrate + ettringite, and E_2 for gypsum + hydrargillite + ettringite. The analytical data of the equilibrium solutions coexisting with the crystalline phases mentioned here

TABLE 1—STABLE INVARIANT POINTS (FOR 20 C) ACCORDING TO J. D'ANS AND H. EICK; THE DATA IN PARENTHESES ARE THOSE OF F. E. JONES FOR 25 C

	Composition of the stable equilibrium solutions									Phases coexisting
	mg per l solution			mg per 2 l solution			Logarithmic coordinates			
Points	Al_2O_3	CaO	$CaSO_4$	Al_2O_3	CaO	$CaSO_4$	Al_2O_3	CaO	$CaSO_4$	
E_2	9.23 (2.55)	17.7 (37.8)	2016 (2040)	18.46	35.4	4032	1.266	1.549	3.606	gypsum, hydrargillite, ettringite
F	2.75 (6.12)	1154 (1076)	1660 (1680)	5.50	2308	3320	0.740	3.363	3.521	gypsum, ettringite, $Ca(OH)_2$
G	2.62 (1.02)	1196 (1063)	14.6 (24.6)	5.24	2392	29.2	0.719	3.379	1.465	hexahydrate, $Ca(OH)_2$, ettringite
H_2	16.17 (9.2)	159.1 (174.6)	35.75 (51)	32.34	318.2	71.50	1.510	2.503	1.854	hexahydrate, hydrargillite, ettringite
b	—	—	2070	—	—	4140	—	—	3.617	gypsum
w	—	1159	—	—	2318	—	—	3.365	—	$Ca(OH)_2$
a_2	<0.9	—	—	<1.8	—	—	<0.255	—	—	hydrargillite
c	—	1105	1683	—	2210	3366	—	3.344	3.527	gypsum, $Ca(OH)_2$
t	3.25	1172	—	6.50	2344	—	0.813	3.370	—	hexahydrate, $Ca(OH)_2$
s_2	25	315	—	50	630	—	1.699	2.799	—	hexahydrate, hydrargillite
j_2	<0.8	—	2064	<1.6	—	4128	<0.204	—	3.616	gypsum, hydrargillite

TABLE 2—METASTABLE INVARIANT POINTS FOR 20C ACCORDING TO J. D'ANS AND H. EICK

	Composition of the metastable equilibrium solutions									Phases coexisting
	mg per l solution			mg per 2 l solution			Logarithmic coordinates			
Points	Al_2O_3	CaO	$CaSO_4$	Al_2O_3	CaO	$CaSO_4$	Al_2O_3	CaO	$CaSO_4$	
j	≈1	—	2064	≈2	—	4128	0.301	—	3.616	gypsum, $Al(OH)_3$, aged 14 days
s	80	190	—	160	380	—	2.204	2.580	—	hexahydrate, $Al(OH)_3$, aged 14 days
m	423.1	481.1	—	846.2	962.2	—	2.928	2.983	—	$4CaO \cdot Al_2O_3 \cdot 13H_2O$, $Al(OH)_3$, aged 14 days
v	3.32	1202	—	6.64	2404	—	0.822	3.381	—	$4CaO \cdot Al_2O_3 \cdot 13H_2O$, $Ca(OH)_2$
e	10.4	8.5	2055	20.8	17	4110	1.318	1.230	3.614	gypsum, $Al(OH)_3$, aged 14 days, ettringite
h	64.5	195.7	11.5	129	391.4	23	2.111	2.593	1.362	hexahydrate, $Al(OH)_3$, aged 14 days, ettringite
n	156.1	335	8	3122	670	16	2.494	2.826	1.204	$Al(OH)_3$, aged 14 days, ettringite, $3CaO \cdot Al_2O_3 \cdot CaSO_4 \cdot 12H_2O$
d	2.22	1179	4	4.44	2358	8	0.647	3.373	0.903	ettringite, $Ca(OH)_2$, $3CaO \cdot Al_2O_3 \cdot CaSO_4 \cdot 12H_2O$

are given in Table 1. Comparison with Jones' results for 25 C shows a very satisfactory general agreement.

The metastable existence of the low-sulfate double salt and of its crystalline solutions with $4CaO \cdot Al_2O_3 \cdot 13H_2O$ was investigated by D'Ans and Eick in a particularly illustrative manner, including the crystallization boundaries of the crystalline solutions. The concentrations of the metastable equilibrium solutions are given in Table 2. A considerable deviation from Jones' results is evident, but the data of D'Ans and Eick solve a series of equilibrium problems which were not satisfactorily accurate in the investigations by Jones.

Another aspect of Fig. 7 is given in Fig. 8, which shows the projection of the system on the plane $CaO\text{-}Al_2O_3\text{-}H_2O$, together with a schematic sketch of the three-dimensional position of the important quaternary equilibrium points. Difficulty in understanding the details of the intricate equilibrium relations of stable and metastable character is overcome by the projection

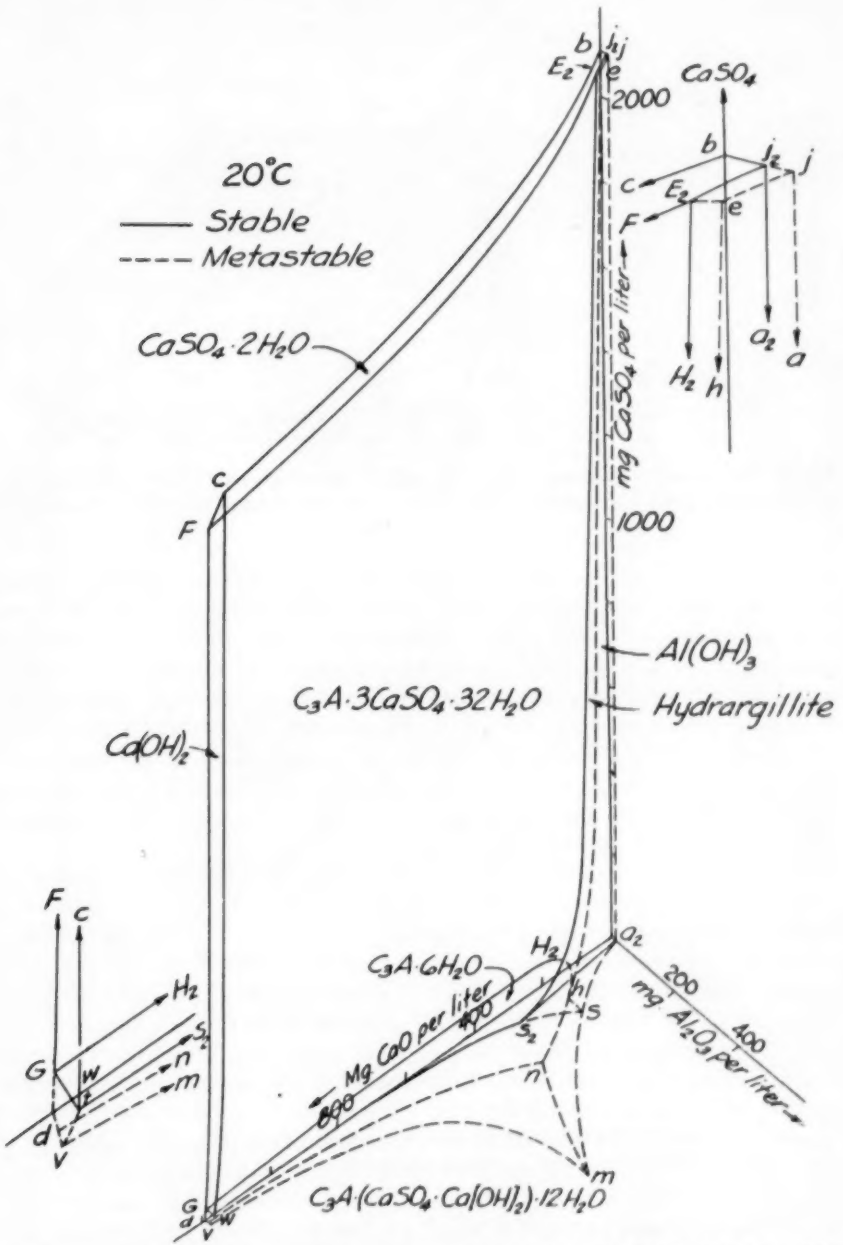

Fig. 7—Phase equilibrium diagram of the system CaO-Al_2O_3-$CaSO_4$-H_2O for 20 C (D'Ans and Eick)

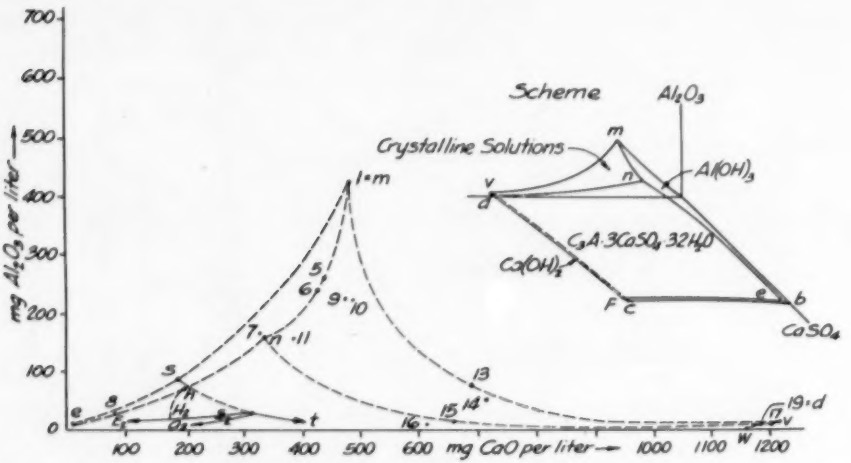

Fig. 8—Same diagram, projected on the plane CaO-Al_2O_3-H_2O (D'Ans and Eick). The smaller figure gives a schematic sketch of the three-dimensional diagram in rectangular coordinates

of the invariant points, field boundary curves, and crystallization surfaces in a logarithmic scale of the three concentration variables. Fig. 9 shows such a projection for the stable crystallization, Fig. 10 for the metastable phenomena. The wide extension of the ettringite field is sufficiently evident, especially its metastable extension to the coexistence boundary d-n with the crystalline solutions of the low-sulfate double salt with $4CaO \cdot Al_2O_3 \cdot 13H_2O$. The aluminate $3CaO \cdot Al_2O_3 \cdot 6H_2O$, however, is suppressed in the metastable diagram; the variability of the precipitation conditions for the alumina hydrogels of variable degrees of aging is clearly indicated in the metastable projection by a few intermediate tie lines.

Concerning the stability fields of ettringite, the metastable crystallization of this double salt, together with the crystalline solutions mentioned above, becomes possible along the boundary curve n-d. Further, the boundary curve m-n characterizes the coexistence of the crystalline solutions with alumina hydrate which is not well-crystallized hydrargillite, but an "aged" cryptocrystalline phase. This product, not otherwise defined, is also in coexistence with gypsum along the boundary curve j-e, and with the hexahydrate along the curve s-h; all these curves are metastable. Ettringite coexists with gypsum along the curve F-e, with $Ca(OH)_2$ along F-d, with "aged" alumina hydrate along n-e. One should note that the solubility of the "aged" hydrate is higher than that of the stable hydrargillite (compare the concentrations for points a_2 and a in Tables 1 and 2), and the same is obvious for points between E and e, or between H and h which may correspond to variable alumina hydrate phases of different degrees of "aging." The intermediate points on the curves mentioned in Fig. 10 correspond to the metastable (aged 14 days) material and other intermediate phases aproxi-

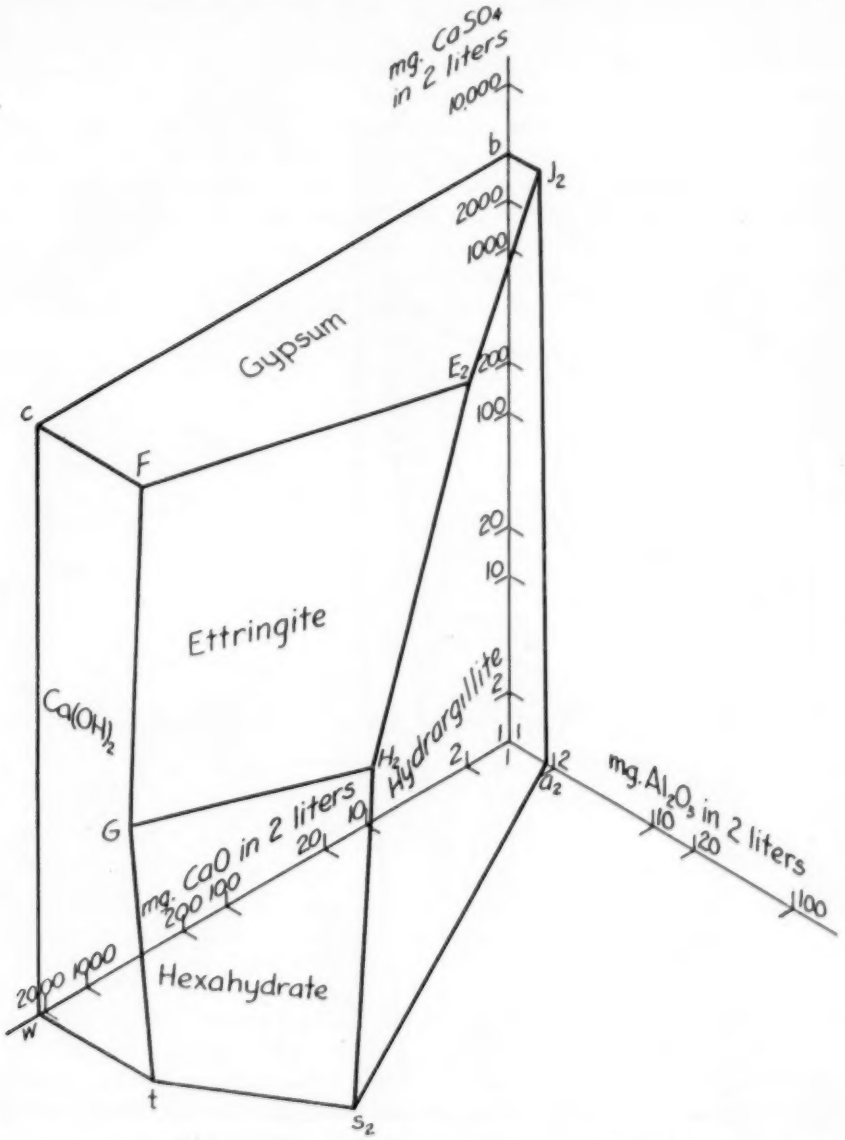

Fig. 9—Stable crystallization diagram for the system CaO-Al_2O_3-$CaSO_4$-H_2O at 20 C, designed with logarithmic coordinates for the concentrations

mating the stable hydrargillite for which the stable equilibria are indicated by dotted curves. Particularly noteworthy is the ternary point m which indicates the coexistence of $4CaO \cdot Al_2O_3 \cdot 13H_2O$ with the aged hydrate of

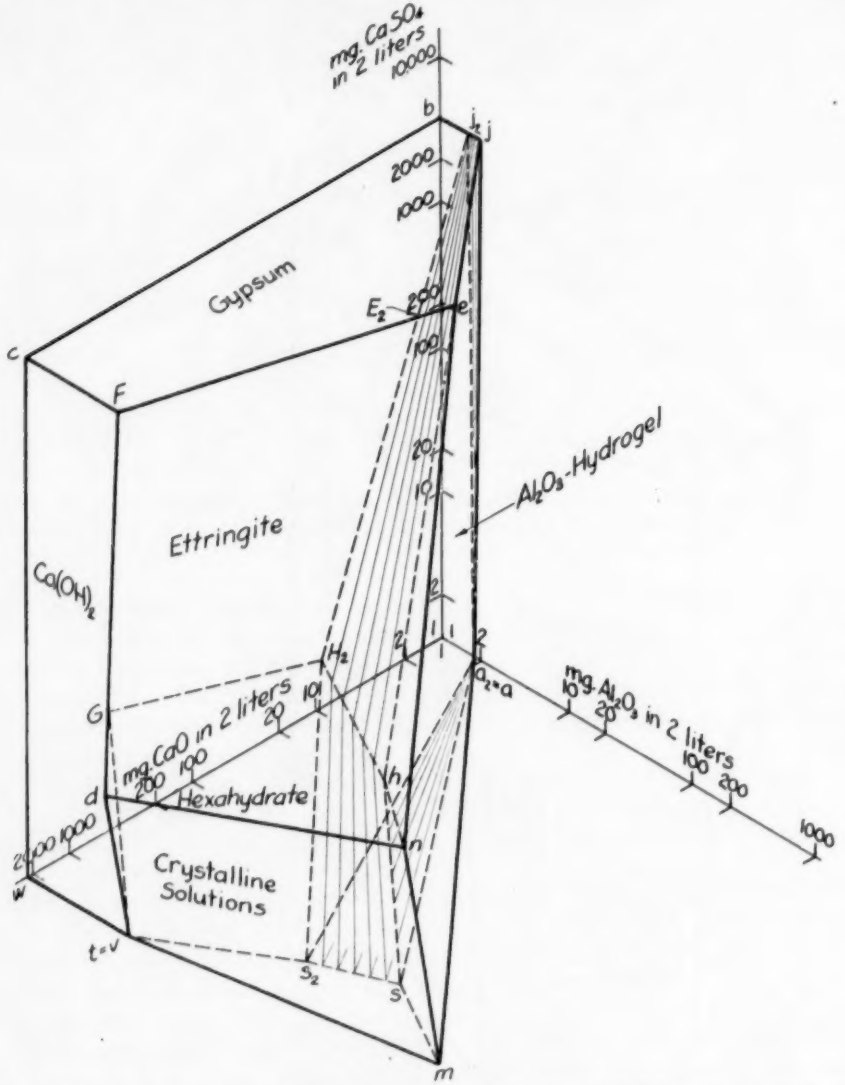

Fig. 10—Metastable crystallization diagram for the system CaO-Al_2O_3-$CaSO_4$-H_2O at 20 C, in logarithmic coordinates of concentrations

alumina, and the point v in which it coexists with $Ca(OH)_2$ as the crystalline phase.

D'Ans and Eick have never observed the crystallization of the calcium aluminate hydrates $2CaO \cdot Al_2O_3 \cdot 8H_2O$, $3CaO \cdot Al_2O_3 \cdot 18H_2O$, and $3CaO \cdot Al_2O_3 \cdot 12H_2O$, in the presence of calcium sulfate in the solution, even after 6

months, while the crystallization of the sulfate aluminates and the crystalline solutions with $4CaO \cdot Al_2O_3 \cdot 13H_2O$, especially the latter, is always very rapid. The formation of crystalline solutions between ettringite and the low-sulfate complex compound, however, was never observed.[15] The latter double salt is replaced by ettringite with increasing $CaSO_4$ content of the liquids. The reaction $3CaO \cdot Al_2O_3 \cdot CaSO_4 \cdot 12H_2O + 2CaSO_4 \rightarrow$ ettringite is completely reversible; the microscopic examination of the crystallized phases gave full evidence of the formation of alumina hydroxide besides the crystalline solutions along the important boundary curve m-n for all the mixes with a ratio $CaO:Al_2O_3$ below 3:1 in $CaSO_4$-richer and of 4:1 in $CaSO_4$-poorer concentrations. Also of interest is a distinct effect of the compositions of the

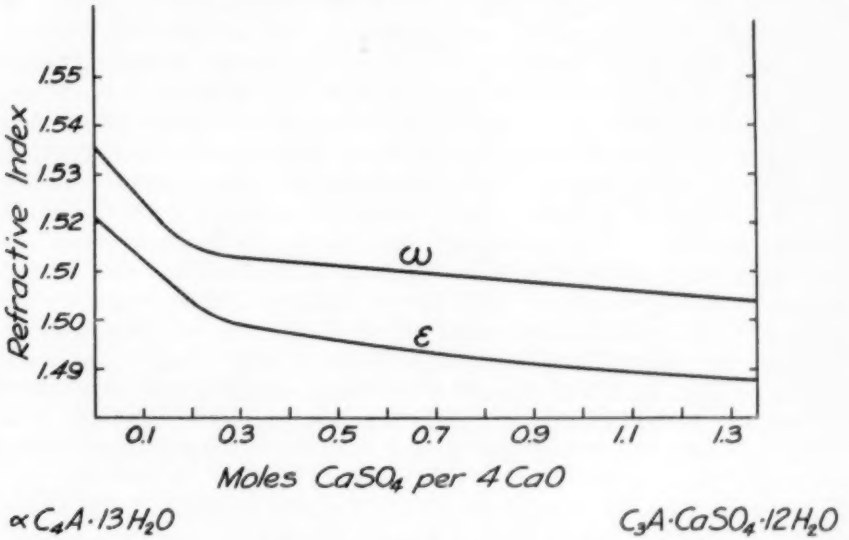

Fig. 11—Refractive indices ω and ϵ for the crystalline solutions of the series $3CaO \cdot Al_2O_3 \cdot CaSO_4 \cdot 12H_2O$—$3CaO \cdot Al_2O_3 \cdot Ca(OH)_2 \cdot 12H_2O$ as a function of their composition. (D'Ans and Eick)

solutions on the crystal habitus of ettringite; in sulfate-richer solutions the crystals are more stout-prismatic or columnar, while in solutions lower in sulfate the well-known slender needle (acicular) type appears. A metastable crystallization of ettringite from supersaturated solutions occurs if a tenth-molar solution of aluminum sulfate is added to a lime hydrate solution with calcium hydroxide in excess. First, a floccular precipitation of an alumina hydrate gel is observed which in a secondary reaction with calcium sulfate is changed to ettringite, evidently combined with a partial neutralization of calcium hydroxide, according to the reaction $2Al(OH)_3 + 3CaSO_4 + yCa(OH)_2 + aq \rightarrow$ ettringite $+ (y - 3)\ Ca(OH)_2 + aq$. This 2-step reaction shows a way for the synthesis of pure ettringite from solutions in which

$CaSO_4$ and $Ca(OH)_2$ must always be in excess to avoid a contamination of the double salt with alumina hydrogel. The pure low-sulfate double salt is only synthesized under very critical concentration conditions; the ratio $CaO:Al_2O_3$ must be higher than 3:1 to suppress the precipitation of alumina hydrogel. The composition of the crystalline solutions is determined from the optical constants of the hexagonal tabular crystals from Fig. 11.

ACTION OF CALCIUM SULFATE SOLUTIONS ON SETTING AND HARDENING CEMENTS

The application of the equilibria of the system $Ca(OH)_2$-$Al(OH)_3$-$CaSO_4$-$Al_2(SO_4)_3$-H_2O to the setting and hardening of hydraulic binders is particularly important in those cases in which the liquid phases present in the mortar or concrete mixes are saturated with calcium hydroxide. This is, of course, the case in setting portland cements and also in alkali-activated blast furnace cements.* Ettringite is formed, *e.g.*, along the curve F-G to the metastable end point d, *i.e.*, together with a slight development of the low-sulfate double salt. Only if very little calcium sulfate is present, the reaction ends in the point v. In the hydration of common portland cements, *e.g.*, with 6 percent Al_2O_3 and 2 percent $CaSO_4$, *i.e.*, with a molar ratio Al_2O_3: $CaSO_4$ of about 1:0.25, ettringite cannot appear in the final phases. On the other hand, the reaction of set portland cement with sulfate-containing waters is always connected with the crystallization of ettringite, if only 4 mg of $CaSO_4$ are contained in one liter of the solution saturated with lime hydrate (in point d of the diagram). On the surface of aged mortars and concretes which have been changed to calcium carbonate by the absorption of atmospheric carbon dioxide, the metastable point n with 8 mg of $CaSO_4$ per liter solution would indicate the critical condition for the crystallization of ettringite in case no free lime hydrate is present.

However, a primarily low concentration of free $Ca(OH)_2$ in the equilibrium solution and, at the same time, a high concentration of calcium sulfate is observed in setting sulfate slag cements. If the calcium sulfate is entirely bound in the reactions, the end product of crystallization will be ettringite together with the low-sulfate double salt and alumina hydrogel (besides the calcium silicate hydrates from the clinker or slag). The representing points would be placed near point n. The slow reaction ettringite → low-sulfate double salt is connected with an increasing concentration of $Ca(OH)_2$ up to about 300 mg per l, at which the precipitation of alumina hydrogel becomes possible.

D'Ans and Eick[13] give a really fundamental and clear basis for the general analysis of every hydraulic binder system on the quaternary system in which calcium sulfate may react either in the setting and hardening process or in the later long-term reactions in mortars and concretes, *i.e.*, with corroding

*Limited space does not permit discussing here the conditions of alkali-activated blast furnace slag cements. It may be sufficient to outline the prevailingly observed formation of $4CaO \cdot Al_2O_3 \cdot 13H_2O$ and of other hydrates either in well-developed crystals, or as reaction hulls around the grains. A relatively high concentration of $Ca(OH)_2$ is required to remove these reaction-impeding layers by dissolution.

sulfate-containing waters. The critical concentrations of only 4 or 8 mg of $CaSO_4$ per l of solution for the crystallization of the expansive ettringite illustrate the imminent danger in exposing hydraulic binders of high basicity to the sulfate in destructive ground waters or to saturated sulfate solutions stagnant in porous systems.

ACTION OF SULFATE SOLUTIONS WITH POZZOLANS

Corrosive sulfate solutions may occur in the interior of surface-active lightweight products which are used on a rapidly increasing scale as modern construction materials. We think here primarily of the lightweight trass and pozzolan-gypsum plaster materials often used to construct walls or ceilings and which, in their typical compositions, bring about exactly the conditions discussed in D'Ans and Eick's investigations. Trass and pozzolans, especially volcanic ashes and tuffs or related rocks in a more or less loose-granular consistency, have in common with the hydraulic binders proper (portland cements or blast furnace slags) the presence of highly activated ingredients among which gamma alumina (in calcined clay), and easily decomposed aluminosilicates or alkalies (in leucite), and alkaline earths (in plagioclase), are the most characteristic compounds. In every rock material in which such activated molecules occur, the contact with more or less concentrated solutions of calcium sulfate will bring about a slower or more rapid crystallization of ettringite that causes a long-term expansion of the products. The most critical conditions, of course, must be expected in products in which gypsum plaster, *i.e.*, the hydration products of industrial stucco (the hemihydrate of calcium sulfate), is the binder material for the trass or pozzolan "fillers," with their contents of activated alumina and aluminosilicates. The high reactivity of the hemihydrate is a particularly important factor; it easily forms during its hydration supersaturated solutions from which gypsum crystallizes in a second step of reaction. On the contrary, anhydrite is very sluggish, not only in its hydration to gypsum, but also in its reactions with calcium aluminates as D'Ans and Eick observed in their most instructive microscopic investigations of such reactions. Stucco immediately forms ettringite with calcium aluminates; anhydrite, on the other hand, reacts because of its sluggishness, in exchange with tricalcium aluminate, first to the low-sulfate double salt.

Activated gamma alumina is the typical ingredient of calcined clay products, *e.g.*, in brick powder and similar "artificial pozzolans," because the meta-kaolinite formed from decomposed kaolinite particles is nothing but a highly reactive and fine granular mixture of gamma alumina and amorphous silica. The natural pozzolans, such as Rhineland trass and Italian volcanic ejectamenta used industrially in hydraulic binders and related products, contain alumina and aluminosilicates in a highly activated state, either by the action of a deliberate thermal treatment (many of the Italian pozzolans are fired to 700 C and quenched)[16] before their industrial use or by the slow disintegration of their feldspar and feldspathoid content under the weather-

ing action of the atmosphere, or even by weak hydrothermal changes (*i.e.*, the formation of base-exchanging "zeolitic" compounds).[17] The fact that in every case the presence of calcium sulfate creates in trass and pozzolanic products the possible formation of ettringite may become a source of trouble. The expansion and "growth" of corresponding industrial products is a striking and deleterious phenomenon.

The rate of such reactions forming ettringite varies greatly; it is high in particularly activated materials and very low in systems in which very sluggish diffusion through inert surface layers may impede the conversion. In concretes, if the surface layers are very dense, the intermediate reaction products of very low solubility or of high swelling capacity will particularly impede the reactions with sulfates. On the other hand, highly porous or foamy fillers will be favorable to the capillary condensation of moisture in the interstices and channels of the structure, and gypsum will be dissolved to form stagnant saturated solutions in the capillaries. The high degree of surface development in the framework of such a product will tremendously enhance the reaction. It depends chiefly, in such porous products, on the rate of base exchange reactions between the zeolitic material and these stagnant solutions, and of course also on climatic conditions. In other words, it will depend on the rate at which the aluminates become available from the filler* to build up the ettringite. Further, from D'Ans and Eick's data it can be concluded that a high concentration in calcium hydroxide is *not* an indispensable condition for the reaction between the aluminates and calcium sulfate. It is very important to know that, in a pozzolan-gypsum body, ettringite is formed only if the critical concentrations in calcium sulfate for the establishment of the equilibrium solutions are maintained.† Such reactions in pores between zeolitic pozzolan + gypsum plaster products and stagnant saturated solutions occur in the neighborhood of the $CaSO_4$-apex of the quaternary diagram given by D'Ans and Eick. Even the very slow formation of ettringite in microscopic samples from a solution in contact with such sluggishly reacting agents, like calcium dialuminate and anhydrite, has been described by these authors. It should not be overlooked that the glassy state of volcanic ashes, tuffs, and blast furnace slags brings about activation of the aluminosilicates, and that this activation may even be increased by special thermal treatments, *e.g.*, the rapid expulsion of the natural gas and water content. Formation of any of the so-called meta-phases†† will increase the reactivity. The reactions of such materials with gypsum plaster correspond, in a first approximation, with those found by D'Ans and Eick for sulfate slag cements, especially if the blast furnace slags are of the low-lime type. The

*The reaction mechanism of this release of aluminates will probably be incongruent.

†This fact was early described by P. Mecke, *Tonindustrie-Zeitung*, V. 54, 1930, pp. 444-446, for "*Si-Staff*," and trass, diatomaceous earth, etc. Later, P. P. Budnikov investigated the abundant crystallization of ettringite in "clay-gypsum cements" with meta-kaolinite as the reactive agent (*Kommunalnoe Stroitelstvo*, 1938, No. 9, pp. 8-14) and "anhydrite-clay cements" (*Stroiteln. Materialy*, 1938, No. 12, pp. 36-39). Also A. Ferrari (*Tonindustrie-Zeitung*, V. 59, 1935, p. 316; *cf. Le industrie costruttive*, V. 5, 1924, p. 45) investigated the rapid absorption of calcium sulfate on a typical pozzolan from the Romagna.

††Meta-phases are defined as fine-crystalline, highly dispersed aggregates formed by dehydration processes from hydrated compounds by the volatilization of volatile ingredients in a calcination process. For zeolitic materials they are evidently intermediate phases, but highly activated products because of their thermodynamic instability and high specific surface (*cf.* the great number of studies by J. A. Hedvall *et al.* on reactions in the solid state).

crystallization of ettringite chiefly occurs along the curve e-n in the equilibrium diagram, *i.e.*, in coexistence of ettringite with alumina hydrate gels which may envelop the reagents and, therefore, slow down the rate of the reaction to a pronounced sluggishness. The expansion observed in such products will be a typical long-term process.

PRACTICAL ASPECTS

In recent years, many observations have alarmed the civil engineering world by indicating instabilities of building materials that, according to the usual experience, should be inert to the action of sulfate waters. These phenomena concern pozzolanic mortars and concretes of the classical Mediterranean type which showed expansion in contact with sulfate solutions, although the same pozzolans have been famous for their resistance to sea-water action since ancient times. Parissi[18] early observed a distinct decrease in mechanical strength and even a later destruction of pozzolanic mortars made with the San Sebastiano (Romagna) tuffaceous material as the "inert" basis. Test samples, after 28 days of curing, showed a strong swelling and fissuring if more than 10 percent of gypsum was added to a pozzolan-lime hydrate mix. The tuffaceous rock is entirely free from clay and also does not contain any organic contaminations. Evidently the addition of gypsum causes the formation of ettringite. A full explanation of this phenomenon was given by Malquori and Spadano,[19] who clearly recognized that activated aluminous material in the pozzolans binds a large amount of calcium sulfate and forms abundant ettringite identified by microscopic methods. The rate of this reaction is determined by the character of the activation of the alumina, either by the natural process of hydrothermal decomposition of aluminosilicates in the tuffaceous rock or by an artificial activation, *e.g.*, the addition of alum or other aluminum-containing substances which are often used as regulators for the setting of stucco plasters.[20] Practically alumina-free mixes do not react with calcium sulfate. If, on the other hand, the solution phase present in the capillary system of the tuffaceous "sponge" is constantly saturated, *i.e.*, in a condensation equilibrium of water vapor from the surrounding atmosphere of variable moisture contents, an ionic reaction occurs in which the excess Ca^{2+} and Al^{3+} cations and the SO_4^{2-} anions form ettringite with its typical expansive properties, even in the absence of free lime hydrate. This fact is highly important in discussing the long-term expansion phenomena which may occur in apparently highly siliceous rocks bonded with gypsum plaster. By a thermal treatment, *e.g.*, the well-known "popping" process mentioned above, the aluminosilicates "dissolved" in the volcanic glass are changed and form highly activated alumina or aluminates.* The presence of lime hydrate will only accelerate the formation of ettringite if the mix shows a more basic composition, *i.e.*, if it is plotted in the phase equilibrium diagram in the wide field of the primary crystallization of ettringite.

*For the rich literature on the chemical character, especially of German trass and Italian volcanic pozzolans see W. Eitel, *Physical Chemistry of the Silicates*, University of Chicago Press, 1954, p. 1255 ff.

This latter case was investigated recently by Turriziani and Schippa[21] in agreement with the early observation of Ferrari that Roman pozzolan fixes considerable amounts of calcium sulfate in the presence of lime hydrate (of a slightly hydraulic lime). Turriziani and Schippa chose for their experiments the pozzolan from Segni, in a fraction finer than 4900 mesh (with 4 percent residue on the sieve with 10,000 mesh), which is an intermediate leucitite-tephrite tuff. The lime hydrate added was 15 to 25 percent, that of gypsum varied between 0 and 15 percent. All samples of the series with more than 10 percent gypsum showed cracking after 28 days of curing. The gypsum fixed after 30 days was about 40 percent, after 90 days 50 percent of its original weight, if the solution phase was kept constantly saturated with calcium sulfate. The x-ray spectrograms of these products show a rich abundance of ettringite; in no case was the low-sulfate double salt identified, only residual gypsum. Particularly important is the hypothesis of Turriziani and Schippa that the first stage of the reactions is an adsorption of calcium sulfate on the surface of the tuffaceous material in which hydrated alumina may also participate in the chemical conversion of the pozzolan. Only in mixes with a low percentage of gypsum it may happen that the low-sulfate double salt is formed as soon as the ratio $CaSO_4$:Al_2O_3 is lower than 3:1 in the adsorption phase. These conclusions are entirely in agreement with the postulates of the equilibrium diagram of D'Ans and Eick outlined above.

It is most remarkable to see how, in portland cement mortars reacting with sulfate solutions, the precipitation of ettringite is very rapid, while in pozzolanic systems the adsorption and later crystallization of ettringite develop fully only over weeks and months. Consequently, the ettringite will be found only in the reaction products of higher siliceous materials, *e.g.*, diatomaceous earths, *Si-stoff*, or in siliceous volcanic glasses like obsidians, if the calcium sulfate is present in stagnant saturated solutions impregnating the whole aggregate, *i.e.*, if it is always present in a large excess. It will then react with activated alumina in these materials if the activation has taken place, *e.g.*, by a thermal treatment (calcination, expansion or "popping" in a flame kiln). It may be expected that such reactions will be observed in the last group of phenomena if the investigations are extended over long periods.[22]

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Discussion of this paper should reach ACI headquarters in triplicate by May 1, 1957, for publication in the Part 2, December 1957 JOURNAL.

Title No. 53-38

Tensile Splitting Test and High Strength Concrete Test Cylinders*

By SVEN THAULOW†

SYNOPSIS

This paper describes an indirect tensile test for determination of tensile splitting strength of concrete and a Norwegian method for preparing cylindrical specimens in which sufficient compaction of the concrete is secured, and capping for compression test is eliminated. This method of making cylinders is particularly valuable for high strength concretes, where thickness and quality of the capping material may have considerably more effect on the compressive strength test results than for lower strength concretes.

TENSILE SPLITTING TEST

There are many technical difficulties in executing a true tensile strength test. A uniform stress distribution which makes it possible to calculate the true tensile strength is difficult to obtain. The method commonly used to determine tensile properties of concrete is the flexural beam test by third-point loading a 6x6-in. beam over an 18-in. span (ASTM C 78-44). The flexural strength is calculated from the bending moment at failure, assuming a straight line stress distribution according to Hooke's law. This is not entirely true, however, and the calculated flexural strength may be about twice as high as the true tensile strength.

The beam test has the advantage that the end pieces of the broken beam can be used to determine the compressive strength of the concrete. These compressive strength results, however, probably differ more from the actual strength of the field concrete than the compressive strength based on standard 6x12-in. cylindrical specimens.

Many attempts have been made to find a substitute for the beam test, and it is possible that the tensile splitting test of a cylindrical specimen may be the solution to the problem (Fig. 1). This method was developed independently in two countries—in Japan by Akazawa[1] and in Brazil by Carneiro[2]—where the methods already are standardized. Investigations of the method are now in progress in various European countries.[2,4,5,6]

When a cylindrical specimen is loaded to failure in the direction of its

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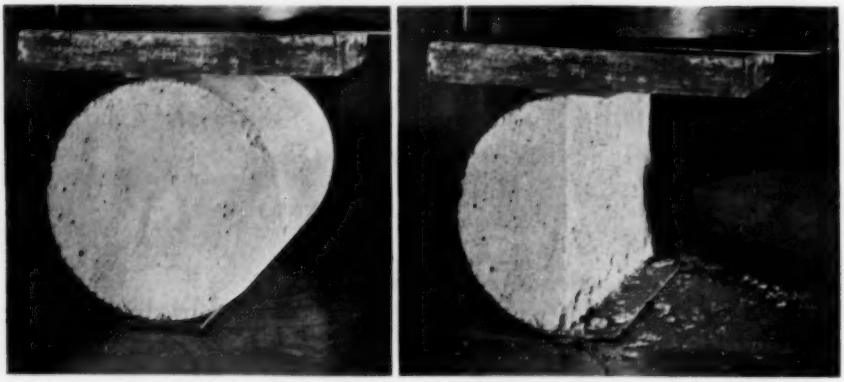

Fig. 1—Loading a 6x12-in. cylindrical concrete (mortar) specimen on its sides permits determination of the tensile splitting strength. Specimen before and after failure

diameter the tensile splitting strength f_t of the concrete can be computed as follows:

$$f_t = \frac{2}{\pi d l} \times P$$

where P is the load at rupture and d and l the diameter and length of the cylinder, respectively. For example, if P = 45,000 lb, d = 6 in., and l = 12 in., the tensile splitting strength is:

$$f_t = \frac{2}{\pi d l} \times P = \frac{2}{3.14 \times 6 \times 12} \times 45{,}000 = 400 \text{ psi}$$

The theory for the stress distribution was first developed by Hertz.[7] Later A. and L. Foeppl,[8] Timoshenko and Goodier[9], Frocht,[10] and Peltier,[3] dealt with the problem and its theory. The theoretical stress distribution is shown in Fig. 2, which assumes that the concrete obeys Hooke's law up to the point of failure. The tensile splitting strength is approximately 60 percent of the calculated flexural strength.

In Fig. 3 the tensile splitting strength is plotted as a percentage of compressive strength according to tests performed by Akazawa,[1] Carneiro,[2] and Efsen-Glarbo.[5] For a concrete with given cement and aggregates there is probably a constant relationship between compressive strength and "split-tensile" strength.

Tests made in Denmark[5] and Norway[12] indicate that the tensile splitting strength is largely independent of length and diameter of the specimen.

In the generatrices* of the specimen which contact the loading surfaces of the testing machine, high compressive stresses are developed. To distribute the load, and reduce variation in strength resulting, a strip of card-

*If the cylindrical specimen is stored horizontally during setting as described later, the cylinder shall be placed in the machine so that the part of the specimen turned upward during setting is now turned to the side.

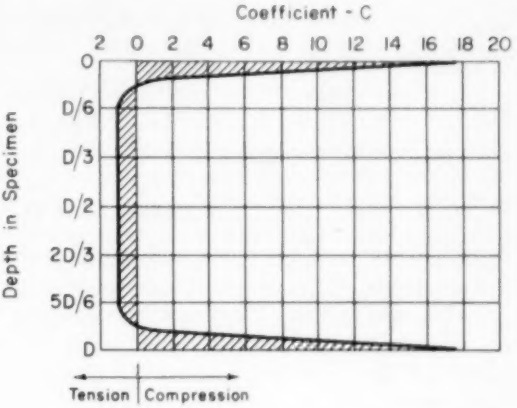

Fig. 2—Stress distribution in the tensile splitting test when width of loading strips is $d/12$.[4] Compressive stress in the generatrices = C f_t, or approximately 18 times the tensile splitting stress

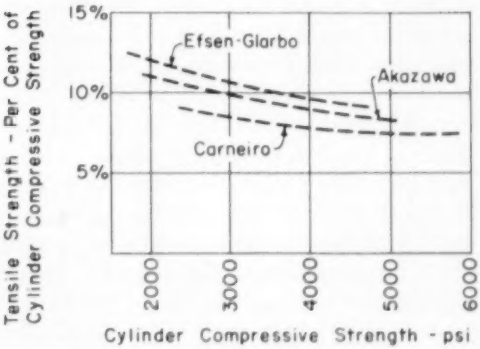

Fig. 3—Tensile splitting strength in percentage of compressive strength. (Tests performed by the author are close to Efsen-Glarbo tests)

board or plywood 1/8 in. thick is recommended between specimen and loading surfaces. If the width of the strips is made 2 in., the placement is simpler in order to obtain necessary alignment. Peltier[3] has pointed out the theoretical advantages of using thin strips instead of thick narrow strips.

The rate of loading could be kept at 1/4 of the rate of loading used for compressive strength test with the same specimen. If this rate of loading is, for example, 35 psi per sec, the rate of loading for the tensile splitting test should be 9 psi per sec.[12] The speed of the testing machine then will be approximately the same for both tensile splitting and compressive strength test.

The advantages of the test method are:

1. Cylindrical specimens and compressive testing machine can be used for determination of both tensile splitting and compressive strengths.*
2. The tensile splitting strength is closer to the true tensile strength than the flexural strength.
3. It is easier, and less expensive, to core a cylindrical specimen from hardened concrete than to cut out a beam. According to experience in Sweden, 25 cylindrical cores can be taken from a pavement daily using diamond coring.

*It can be mentioned that if a split tensile tested cylinder is loaded afterward for compressive strength the loss in compressive strength in relation to an unsplit cylinder usually will be less than 10 percent.[12] The tensile splitting test must then stop immediately when failure occurs.

4. The tensile splitting test may be less sensitive to small cracks occurring in the exterior of the specimen than the beam test, because the exterior parts of the cylindrical specimen are in compression (Fig. 2).

The disadvantage of the test in relation to the beam test is that flexural strength is used as a basis for design of concrete pavements. The relationship between flexural strength and tensile splitting strength should be investigated.

COMPRESSION TESTING

In Europe both cylinders and cubes are commonly used for compression tests. There is no constant relationship between compressive strength of cylinders and cubes. The size of specimen, grading of aggregates, and quality of concrete have a marked influence on the relationship.[12-18]

Neither the cylinder nor the cube gives a precise picture of the strength of the concrete in the structure, but it is generally agreed that the cylinder strength is closer to it than the cube strength. The apparent strength of cubes is increased by friction in loading surfaces between specimen and testing machine. This influence is almost eliminated when the height-width ratio of the specimen is 2, as in the 6x12-in. cylinder.[13,14,15]

One advantage with the cubes cast in steel molds is that they can be tested for compressive strength without capping the loading surfaces, when the load is applied against the sides of the cubes which are placed against the steel. Cylindrical specimens, however, have to be capped or ground because the loading surfaces must be even and parallel.

With the increase of high strength concretes for prestressed concrete, precast elements, concrete pipes, and other uses, the capping materials and methods have assumed increasing importance. Investigations by Troxell,[19] Gonnerman,[20] Kennedy,[21] and Nycander,[22] have shown that the thickness and the quality of the capping material have an important influence on the compressive strength results, especially for high strength concretes. A good

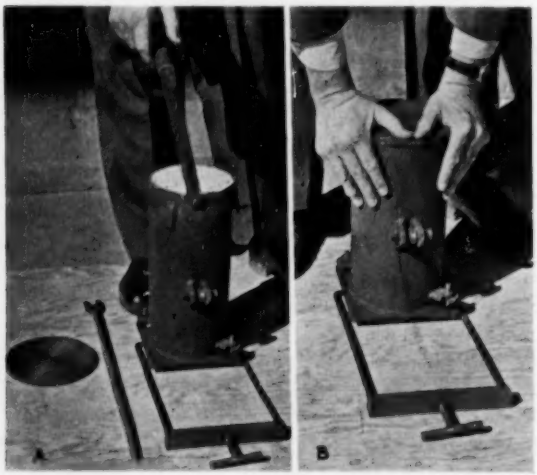

Fig. 4 (A)—After filling and rodding, the concrete is further compacted by letting the handle of the collar fall freely against the sides of the cylinder, alternately on each side. (B), a top plate is worked down with sliding motion until the plate fits snugly against the steel mold

capping material ought to have the same compressive strength and the same modulus of elasticity as the concrete in the specimen. This is difficult to achieve when using material other than the concrete itself.

The following method, using specially designed molds (extensively and successfully used in Norway and other countries during the last four years) eliminates the need for capping. See reference 12 for detailed description.

The concrete is placed in the mold and the collar in three layers, and each layer is rodded 25 times. Then the concrete is compacted further by letting the handle of the collar fall freely against the sides of the mold, alternately on each side (Fig. 4A). The number of blows used is a function of the previously measured consistency. If the consistency is measured with the concrete tester,[12] the number of blows for the compressive test specimen shall be half the number of blows measured by the consistency tests.* Tests[12]

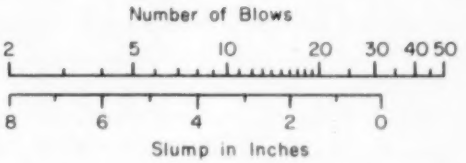

Fig. 5 — Number of compacting blows for cylindrical specimen as a function of consistency measured with the slump cone

have shown that this is sufficient to give the specimen complete compaction. More blows do not increase unit weight and compressive strength results.

If the consistency is measured with the slump test, Fig. 5 can be used for determining the number of compaction blows. For very dry mixes with "minus zero" slump, measuring of consistency and compaction of cylindrical specimens is accomplished with a drop table.[12]

After compaction the collar with the handle and most of the concrete within it is removed. A little surplus material is left on the top and the top plate is worked on to the top with a sliding and rotating movement until one can hear and feel that the plate fits snugly against the end of the mold (Fig. 4B). The top plate is then secured with the stirrup and the specimen stored horizontally with the split in the cylinder uppermost until demolding. The top and bottom plates are planed, and they are so thick and rigid that the loading surfaces of the specimens afterwards are absolutely even.

Because of horizontal storage during the setting of the concrete, the applied load in the compression test is perpendicular to the direction of settlement, but there has been no indication that this has any detrimental effect. (Cubes are also tested perpendicular to the direction of settlement.)

In the case of very lean mixes, bleeding may occur under the coarsest particles of the concrete and reduce the strength in relation to vertically stored specimens. On the other hand, most structures with bending stresses are cast vertically and are stressed horizontally.

The settling of the concrete reduces the cross section of the specimen. This effect is very small in rich mixes. The cross section can easily and accurately

*A description of the apparatus and the method for measuring consistency with the concrete tester also appeared in the ACI JOURNAL, V. 25, No. 7, Mar. 1954, News Letter pp. 10-11, 24, 25.

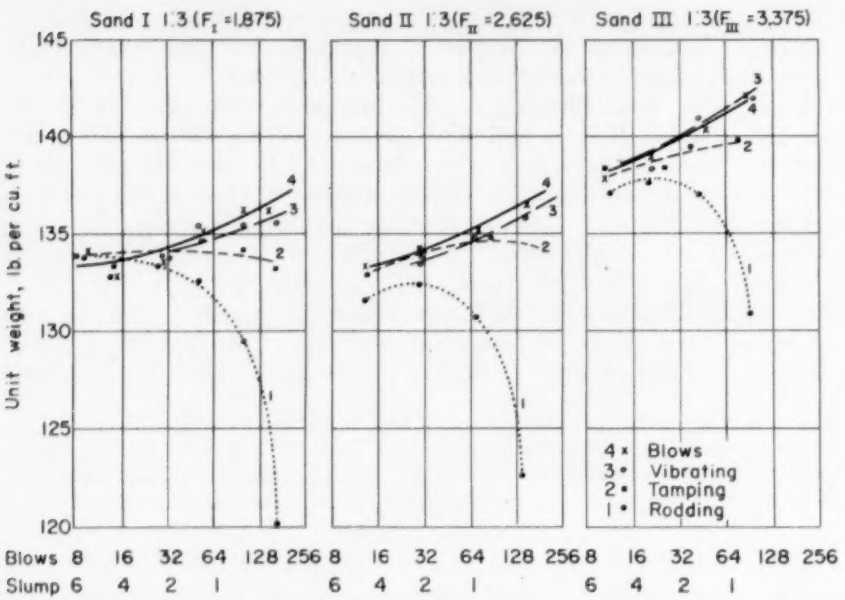

Fig. 6—Unit weights of 1:3 mortar with sand of different grading (fineness modulus F = 1.875, 2.625, and 3.375) by different consistencies, compacted in four different ways in the 10-liter concrete tester:

1. **Rodding according to ASTM, three layers, each rodded 25 times**
2. **Tamping with a 3.5-lb tamper, dropped 6 in. Three layers, each tamped 25 times**
3. **Vibrating on a "Vebetable" until concrete appeared wet and smooth on the surface**
4. **Blows. Unit weight of concrete remolded in the concrete tester after a consistency test**

be determined by measuring the water which is displaced when the specimen is immersed in water.

Paraffined cardboard molds are in common use at the present. Tests made by the U. S. Bureau of Reclamation,[23] Burmeister,[24] and Howard[25] showed that paper and cardboard molds gave about 3.5, 9.5, and 13.3 percent, respectively, less strength than when steel molds were used.

The coefficient of variation in strength results in actual field work ranges from 10 to 25 percent and even more. The average strength of the concrete has to be sufficiently high so that theoretically at least 90 percent of all compressive strength tests are above required minimum strength.[26]

High variations in strength are undoubtedly in part a consequence of errors and carelessness during the testing procedure. A reduction in the coefficient of variation for the strength test means lower required average strength and a saving in cement cost which will, in many cases, more than pay the expense of better control.

Principal advantages of the horizontal casting are:

1. The compaction of the specimen is based on the measured consistency and is independent of the personal judgment of the operator. Tests[12] have shown the importance of compaction. Fig. 6 shows the test results for unit weight determinations of 1:3 mortar compacted in four different ways. For fresh concrete with slump less than 2 in., the unit weight, and hence the strength of the concrete, may drop due to insufficient compaction by the rodding method alone.
2. Specimens can be kept in the moisture room or water tank until testing. They need not be taken out for capping, during which operation drying may influence compressive strength results. If desired the molded specimen may also be stored under water immediately after concreting until demolding after 24 hr.
3. No capping is required because top and bottom are smoothly cast in contact with ground steel plates.
4. Loss of water after molding is negligible because the split in the mold is uppermost.

CONCLUDING REMARKS

In the development of concrete and concrete technology the use of international testing methods is of great importance.

The method for determining the tensile splitting strength of concrete ought to be further examined. The advantages and disadvantages of the test compared with the beam test should be discussed thoroughly.

The method for compacting specimens according to the measured consistency of the concrete, and the horizontal storing which eliminates capping are proposed as an improvement of the universally used compressive cylinder strength test.

In a recent paper H. Rüsch[27] has compared cubes and cylinders for compressive strength tests. His conclusion is that cylinders are superior, and that two decisive developments—the elimination of capping and the tensile splitting strength test—have made it important again to discuss accepting the cylindrical specimen as a universal standard.

If cylindrical specimens should be commonly accepted for both compressive and tensile strength determination, it ought to be considered whether testing of cement also should follow the same outline. Brazil and Uruguay both use 2x4-in. cylindrical specimens for compressive strength tests.[28] Good correlation between cement- and concrete-test results is of great importance.

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Discussion of this paper should reach ACI headquarters in triplicate by May 1, 1957, for publication in the Part 2, December 1957 JOURNAL.

CONCRETE BRIEFS

Notes from Field and Office

Influence of Edge Condition in Flat Reinforced Concrete Shell-Dome 707

Influence of Edge Condition in Flat Reinforced Concrete Shell-Dome

By MARTIN SCHULZ*

Fig. 1—School of Agriculture auditorium, El Salvador

A reinforced concrete dome (Fig. 1), designed by architects Karl Katstaller and Ehrentraud Schott, roofs the auditorium of the School of Agriculture in El Salvador. The dome has a diameter of 28 m (91.9 ft) and a thickness of 6 cm (2.4 in.). A circular beam was placed at the edge of the dome to take the tension forces, torque moments, and bending moments. The ring was supported every 4 m (13.1 ft) by columns. Dimensions of the shell are given in Fig. 2.

The following article shows the influence of the perturbation on the membrane stresses.

Calculations for the shell can be divided into two parts: (1) membrane condition, where the thickness of the shell is assumed small compared to the other dimensions of the shell and the radii of curvature; and (2) edge condition.

The stresses calculated from these two conditions have to be combined. The notation of the forces and moments are as follows: N_ϕ = forces in meridian direction; N_θ = forces in ring direction; and M_ϕ = moments in meridian direction.

Membrane condition

The derivation of the following formulas are found in reference 1. For dead load:

$$N_\phi' = -\frac{g\,a}{1+\cos\phi}$$

$$N_\theta' = -g\,a\left(\cos\phi - \frac{1}{1+\cos\phi}\right)$$

For a = 29.70 m (97.5 ft) and g = 0.0256 kg per sq cm (0.364 psi), the values for N_ϕ' and N_θ' for different values of ϕ are drawn in Fig. 3.

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*Member American Concrete Institute, Structural Engineer, Ministerio de O.O.P.P., Departamento Arquitectura, San Salvador, El Salvador.

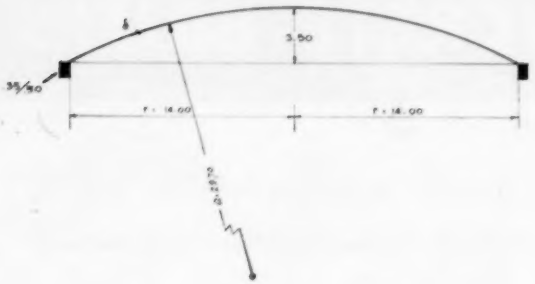

Fig. 2—Shell dimensions

Note: All dimensions and quantities in drawings are in metric system

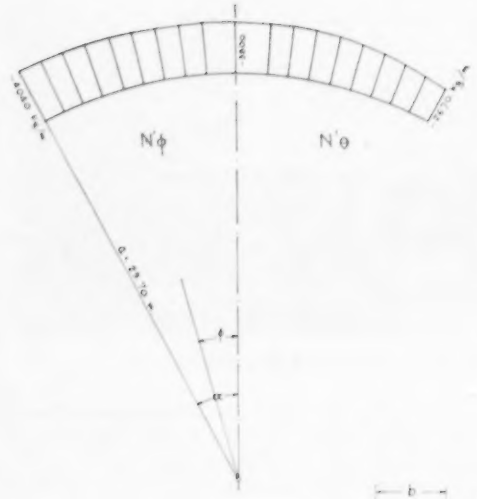

Fig. 3—Values for N_ϕ' and N_θ'

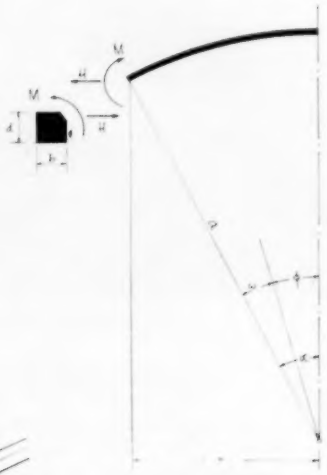

Fig. 4—Edge forces

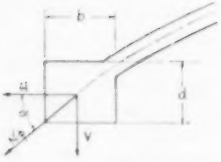

Fig. 5

Edge condition

By the membrane condition the edge of the dome and the ring would have different deformations. The necessary harmony of them, which is conditional on the monolithic connection, is forced by edge forces H and M (Fig. 4) which are uniformly distributed along the edge. The deformations of the shell edge and ring due to the membrane condition are:

$$EA = \frac{r(N_\theta')_{\phi=\alpha}}{2h}$$

$$EB = \frac{1}{2h}\left[-\frac{dN_\theta'}{d\phi} + \mu\frac{dN_\phi'}{d\phi} + (1+\mu)(N_\phi' - N_\theta')\cot\phi \right]_{\phi=\alpha}$$

$$EA' = \frac{\overline{H}\, r^2}{bd}\,; \quad EB' = 0$$

where EA is the displacement and EB the rotation of the edge of the shell. $E\,A'$ is the

displacement of the ring and EB' the rotation of the ring. In this case the latter is zero because the forces act through the center of the ring (see Fig. 5). Also μ = Poisson's ratio, E = modulus of elasticity of concrete.

The deformations of the shell edge and ring due to the edge force H are:

$$EA_H = \frac{H x r \sin \alpha}{h}$$

$$EB_H = \frac{Hx^2 \sin \alpha}{h}$$

$$EA_{H'} = -\frac{4Hr^2}{bd}$$

$$EB_{H'} = -\frac{6\,Hr^2}{bd^2}$$

where EA_H is the displacement and EB_H the rotation of the shell edge. EA_H is the displacement of the ring and $EB_{H'}$ the rotation of the ring.

The deformations of the shell edge and ring due to the edge moment M are:

$$EA_M = \frac{Mr^2 \sin \alpha}{h}$$

$$EB_M = \frac{M\,a}{Kx}$$

$$EA_{M'} = \frac{6\,M\,r^2}{b\,d^2}$$

$$EB_{M'} = \frac{12\,M\,r^2}{b\,d^3}$$

$$\text{where } K = \frac{2E\,h^3}{3(1-\mu^2)}$$

where EA_M is the displacement and EB_M the rotation of the shell edge. $EA_{M'}$ is the displacement of the ring and $EB_{M'}$ the rotation of the ring.

The edge condition of the shell, loaded and supported symmetrically with respect to their axis, is:

$$EA + EA_H + EA_M = EA' + EA_{H'} + EA_{M'}$$
$$EB + EB_H + EB_M = EB' + EB_{H'} + EB_{M'}$$

The moments and forces in the shell produced by $H = 1$ and $M = 1$ are:

$$M_{\phi H=1} = \frac{H\,a}{x}\,e^{-x\omega} \sin \alpha \sin x\omega$$

$$M_{\phi M=1} = Me^{-x\omega}(\sin x\omega + \cos x\omega)$$

The final moments

$$M_\phi = M_{\phi H} + M_{\phi M} \text{ (see Fig. 6)}$$

$$N_{\theta H=1} = 2Hxe^{-x\omega} \sin \alpha \cos x\,\omega$$

$$N_{\theta M=1} = \frac{2M\,x^2}{a}\,e^{-x\omega}(\cos x\,\omega - \sin x\,\omega)$$

The superposition of these two forces $N_{\theta H}$ and $N_{\theta M}$ produced by H and M are

$$N_{\theta HM} = N_{\theta H} + N_{\theta M}$$

where

$$x = \sqrt[8]{\frac{a}{h}}\sqrt[4]{\frac{3}{4}(1-\mu^2)} \text{ (fade away number)}$$

Because of their insignificant values it is unnecessary to analyze the forces $N_{\phi H}$ and $N_{\phi M}$ and moments $M_{\theta H}$ and $M_{\theta M}$.

Example

For α = 28 deg 06 min; r = 14 m (45.9 ft); $\delta = 2h$ = 6 cm (2.4 in.); $\overline{H} = N_\phi \cos \alpha$ = 35.6 kg per cm (2388 lb per ft); b = 35 cm (13.8 in.); d = 50 cm (19.7 in.); x = 29.3; μ = 0; E = 210,000 kg per sq cm (2,986,200 psi); we find the following values for H and M:

M = − 1.09 kg-cm per cm (2.4 ft-lb per ft)
H = + 4.22 kg per cm (283.7 lb per ft)

Fig. 6 shows the moment diagram for M_ϕ and the stress diagram for $N_{\theta HM}$ due to the edge forces H and M. It is seen that the moments and forces reduce quickly in form of damped waves. To find the end values N_θ we have to combine the values found by the membrane condition and the edge condition.

$$N_\theta = N_{\theta'} + N_{\theta H} + N_{\theta M}$$

$$M_\phi = + M_{\phi H} + M_{\phi M}$$

The portion of the membrane condition for the moment M_ϕ is naturally zero. Fig. 7 shows the final stresses.

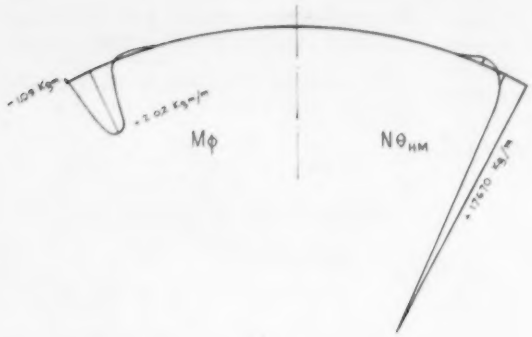

Fig. 6

The final stresses N_ϕ are identical with the membrane forces N_ϕ' because as said before $N_{\phi H}$ and $N_{\phi M}$ are negligible.

It can be seen that boundary effect takes place only near the ring, but because of its large value has to be analyzed carefully. This is important especially in concrete structures where we assume that the concrete does not take any tension and the steel has to be designed for all tension.

The calculation was repeated for the condition of a hinge connection between shell and ring. This analysis was easily performed by using only the first of the two equations established for the edge condition. H is determined and the moments M_ϕ and forces N_θ calculated in the same way as before except that the final moments and forces are:

$$N_\theta = N_\theta' + N_{\theta H}$$

$$M_\phi = M_{\phi H}$$

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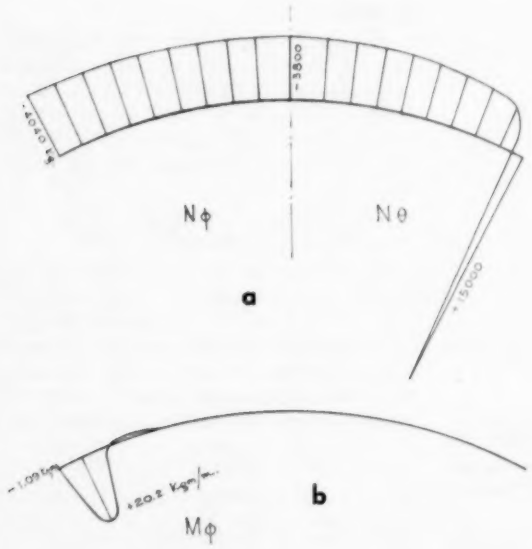

Fig. 7—Final stresses

CURRENT REVIEWS

of Significant Contributions in Foreign and Domestic Publications

Dams

Effect of pore pressures on stresses in gravity dams

O. C. ZIENKIEWICZ, *Proceedings*, ASCE, V. 82, PO4, Aug. 1956, pp. 1042-1 to 1042-16

AUTHOR'S SUMMARY

In this paper the question of stresses arising in a gravity dam due to the action of pore water pressures is considered. Although for a particular type of pressure distribution the answer to this problem has been derived, the author seeks to provide a solution for cases, often existing in practice, where the restrictive assumptions of the known solution do not apply.

In particular, a case of an idealised, triangular shaped dam is analysed, and a general elastic solution obtained. This solution is valid for any arbitrary type of pressure variation. Numerical answers are given to a special case, where a line of drains placed inside the dam reduces the pore pressures discontinuously. Some simple general conclusions are drawn from the results of above computations.

Design

Theory of elastic thin shells (Teoriya uprugiye tonkiye obolochec)

A. L. GOLDENWEIZER, Gosudarstvennoe Izdatelstvo Tehnico-Teoreticheskoe Literaturoy, Moscow, 1953, 544 pp., $3 (23 rubles 50 copecks)

Reviewed by GUNHARD AESTIUS ORAVAS

This book is divided into five major chapters. Chapter I develops the general theory of curved surfaces by the use of generalized curvilinear coordinates. The differential geometry of the surface is treated with the usual application of vector concepts. This part of the subject matter is thoroughly handled, and in addition the basic hypotheses and the general elasticity of the theory of shells are extensively covered. At the end of the chapter the general statical-geometrical relations and the fundamental thin shell equations are developed in terms of generalized curvilinear coordinates.

Chapter II deals with the theory of shells with negligible transverse stiffness (membrane shells). Shells of zero curvature, open and closed spherical shells subjected to various loading conditions, and other noncircular shells are treated with the application of complex functions. The application of complex functions is followed throughout the book. Other more generalized shell shapes are discussed in the light of the previous development.

Chapter III is devoted to the analysis of closed or sectional circular cylindrical shells. Various approximations are worked out and boundary effects thoroughly investigated. Approximations are accompanied by qualitative mathematical motivations.

Chapter IV contains analysis of stressed state of generalized thin shells. Asymptotic methods of solution are investigated and their applicability to generalized thin-shell analysis demonstrated. The boundary effect of thin shells is again investigated by the use of asymptotic integration.

Chapter V contains the approximate methods of analysis for thin shells with arbitrary shapes. Such methods are developed for membrane shells, generalized thin shells and boundary effects. An extensive treatment is given for cylindrical and conical shells at the end of the chapter.

Proficient use has been made of complex

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functions and every chapter has been supplied with numerous practical examples. A lot of material contained in this book is the original contribution of the author and other Russian writers on shells.

It must be said that the author has handled his mathematics in a manner which should be readily understandable for engineers with regular college mathematics background. A proper balance has been kept between mathematical rigor and practical usefulness of the subject matter throughout this monograph. Contrary to the tendency of recent Russian works on the subject to be extremely theoretical, this volume has been written for the shell designer.

The subject matter in this volume is largely drawn from the contemporary Russian work on shells, and some limited use has been made of recent Western contributions on thin shells; however the bibliography given on shell literature is devoid of references to Western sources of information.

In the reviewer's opinion this book represents the best comprehensive non-tensorially notated treatise on shells to date. It is a considerable loss to the Western engineer not to have a translation of this excellent monograph available.

General equations of shells (Allgemeine Schalengrundgleichungen)

Wilhelm Fuchssteiner and Alois Schader, *Beton- und Stahlbetonbau* (Berlin) V. 51, No. 7, July 1956, pp. 145-153

Reviewed by Valdis Lapsins

Theoretical foundation of a generalized theory of shell design presented with numerous mathematical equations. It deals with setting up equations for equilibrium and deformation of any form of shells.

Theory of prestressed concrete design

H. J. Cowan, MacMillan & Co., Ltd., London, and St. Martin's Press Inc., New York, 1956, 264 pp., $8.25

Presents the mathematical theory for the design of statically determinate prestressed concrete elements. It is largely based upon original material by the author, developed as a course of lectures on prestressed concrete design for undergraduate and graduate use. Particular emphasis is placed upon direct design as contrasted to reviewing an assumed design. The book presents British practice better than American, particularly in regard to pre-tensioned products.

Two-way slab design simplified by tabulation

F. R. Hearte, *Civil Engineering*, V. 26, No. 9, Sept. 1956, p. 72

Develops a tabulated procedure for office use in design of two-way slab floors in accordance with the ACI Building Code.

On some extensions of the theory of rigid-jointed folded plate structures with prismatic, narrow individual plates (Ueber einige Erweiterungen der Theorie des steifknotigen Faltwerkes mit prismatischen, schmalen Einzelscheiben)

H. Aichinger, *Der Bauingenieur* (Berlin), V. 30, No. 11, Nov. 1955, pp. 397-403

Reviewed by Aron L. Mirsky

Analytical treatment of structures composed of flat plates. Extension to the case of three plates meeting at a single edge is outlined.

Author's application of "folded plate" methods is thus quite wide, covering slabs on multiple deep-but-narrow girders and even arches or shells, which can be approximated by a series of chords which form the folded plates.

Contribution to the dynamics of sprung foundations for machines (Beitrag zur Dynamik der gefederten Maschinengruendung)

K. Luerenbaum, *VDI Zeitschrift* (Duesseldorf), V. 98, No. 18, June 21, 1956, pp. 976-980

Reviewed by Aron L. Mirsky

Emphasizes analytical and graphical (Mohr's circle) solution of the frequency equation, especially for systems with two planes of symmetry (three degrees of freedom). A series of seven photographs presents views of a model vibrating under various conditions.

Analysis of continuous girders on elastic supports (Berechnung des Durchlauftraegers auf elastisch senkbaren Stuetzen)

G. Worch, *Der Bauingenieur* (Berlin), V. 30, No. 11, Nov. 1955, pp. 388-392

Reviewed by Aron L. Mirsky

Presentation and comparison of method of W. T. Koiter (*De Ingenieur*, V. 55, 1940) and slope-deflection ("Drehwinkelverfahren") method.

Factors affecting vertical loads on underground ducts due to arching

N. C. Costes, *Bulletin* No. 125, Highway Research Board, Sept. 1956, pp. 12-58

Presents a theory of earth pressure on underground conduits. Develops pressures resulting from nearly every combination conceivable as a result of location or construction practices. In addition to the mathematical development, conclusions should be useful both to field personnel and specification writers. Fifty-eight references are provided.

Failure of concrete under compound stress

A. J. Harris, *The Engineer* (London), V. 201, No. 5235, May 25, 1956, pp. 569-570
Reviewed by Aron L. Mirsky

Brief exposition of Coûard's theory of rupture of concrete. Theory, first applied to soil mechanics (*c.f.*, *The Engineering Journal*, Sept. 1955, pp. 1207-1210; "Current Reviews," ACI Journal, Apr. 1956, *Proc.* V. 52, p. 908), uses two parameters, the cohesion and the angle of internal friction. Sources of information relative to the evaluation of these parameters are discussed.

Materials

Synthetic admixtures to concrete (Met kunststof gelegeerd beton)

G. J. Hamer, *Cement* (Amsterdam), V. 8, No. 13-14, Feb. 1956, pp. 327-330
Reviewed by John W. T. Van Erp

An extensive review of the use of synthetic chemical compounds as concrete admixtures, their effect on concrete, and concrete properties at various proportions of admixture. The most widely used admixture is a dispersion of polyvinyl acetate, and its function is essentially to coat all crystalline particles in the concrete and form semi-elastic hinges between them. This explains the deformation capacity and the crack resistance of such concrete. Other properties of such concrete are (1) excellent bonding to old concrete, a reason why it can be used in layers as thin as a fraction of an inch; (2) wear-resistance; (3) good elasticity even at low temperatures; (4) increased flexural and compressive strength. Some applications are road repair, airport landing strips, floors, railroad ties, bridges, and storage tanks. Concrete wearing courses as thin as ¼ in., bonded to old concrete, have been used successfully.

Concrete additives at low temperatures (in Dutch)

P. W. Scharroo, *Cement* (Amsterdam), V. 7, No. 9-10, Oct. 1955, pp. 235-236
Reviewed by John W. T. Van Erp

These additives work in two ways: (1) lowering of freezing point of the mix, (2) accelerating setting time and at the same time increasing the heat of hydration of the cement. Various additives are sodium chloride, soda ash, magnesium chloride, and calcium chloride, of which the last has the most desirable properties. It is used in an amount of 2-4 percent by weight of cement. Increased shrinkage is liable to occur, especially in the beginning. Another good method is that which is extensively used in Russia, utilizing a solution of $CaOCl_2$.

Aluminous cement in winter time construction

C. L. Chapin and W. A. R. Lemire, *Ontario Hydro Research News* (Toronto), V. 8, No. 2, Apr.-June 1956, pp. 9-12, 24

Compares temperature and strength-gain data of aluminous cement concrete and high-early-strength portland cement concrete accelerated with 2 percent calcium chloride. Concludes that there are advantages in the use of aluminous cement concrete in the field conditions encountered which range down to − 50 F, but that the peculiarities in handling the aluminous cement concrete must be taken into account.

How to save cement by proper grading of aggregate

William Grant, *Concrete*, V. 64, No. 9, Sept. 1956, pp. 28-32

Discusses the requirements of fine aggregate for use in concrete masonry, including simple tests to determine their suitability and required processing. Particular attention is given to the problems in proportioning aggregate by volume caused by varying moisture and consequent bulking. A practical discussion intending to aid concrete masonry operators to use available materials to best advantage.

Concrete work in winter time without heating materials and concrete (in Rumanian)

V. M. Medvedjev and G. A. Siso, *Constructivlor si a Materialelor de Constructii* (Bucharest), V. 6, No. 1, Jan. 1955, p. 64

Reviewed by J. J. Polivka

Some of generally used methods, such as surface heating by steam or electricity, casting concrete in enclosed rooms, are compared with methods based on admixture of chemicals, especially of calcium and sodium chloride. Tests in Russian laboratory for the Volga-Don canal showed the following percentage of those chemicals admixed to water, for various freezing temperatures (measured as average during 7-10 days), to be most effective:

Average temperature, deg C	−5	−10	−20
Calcium chloride admixture, percent	6	12	18
Sodium chloride admixture, percent	2	3	4.5

Pavements

Joint spacing in concrete pavements

Research Report No. 17-B, Highway Research Board, Aug. 1956. 159 pp., $2.70

Includes 10-year reports on six experimental projects in various states. The six reports are introduced by an analysis of all the data by E. C. Sutherland, Bureau of Public Roads. The projects were constructed in California, Kentucky, Michigan, Minnesota, Missouri, and Oregon, and included plain and reinforced pavements, and various spacings of both contraction and expansion joints. This report follows earlier progress reports published by the Highway Research Board on these projects.

Use of indirect tensile tests for quality control of pavement concrete

L. L. Simon, *Constructional Review* (Sydney), V. 29, No. 8, Aug. 1956, pp. 23-29

Includes an account of the use of the so-called "Brazilian test" for quality control of field construction of pavement. Comparative tests of compressive strength and flexural strength (as shown by ordinary beam tests) were made and results are included. It is concluded that "quality control of concrete pavements can be based upon a specification which requires the concrete to have an ultimate indirect tensile strength—if tested in accordance with the method of testing recommended—equal to two-thirds of the ultimate flexural strength upon which the pavement design was based." Includes as an appendix a proposed standard method of test for indirect tensile strength of molded concrete cylinders.

Precast Concrete

Cement roof tiles (in Dutch)

P. Groenhart, *Cement* (Amsterdam), V. 7, No. 9-10 and 11-12, Oct. and Dec. 1955, pp. 237-240, 282-285

Reviewed by John W. T. Van Erp

Roof tiles have traditionally been made from clay and only recently concrete roof tiles have been used on an increasing scale, which is due to various improvements in fabricating technique. They are now being made in various colors and also in different surface textures. It seems that thereby the main objection to their use—the drab color and surface texture—has been overcome. Article describes the completely automatic manufacturing methods used.

Survey of concrete transmission line poles

C. F. C. Crisp, *Constructional Review* (Sydney); V. 28, No. 8, Dec. 1955, pp. 20-31; V. 29, No. 1, Jan. 1956, pp. 27-32

A collection of the existing knowledge on concrete transmission poles both ordinary reinforced and prestressed. Hollow, circular, solid rectangular, and shaped sections are mentioned and supplemented in the appendices. The first part of the paper deals with the design of poles for given loadings. In the last part of the paper a comparison is made between timber and concrete poles based on the annual cost. For the cost figures shown, concrete poles have an economic advantage. The superior quality in prestressed poles, especially as regards recovery after overload, are also discussed. Particular credit is paid

to the Florida Power Corp. which supplied the data on the prestressed concrete transmission poles developed and used by them.

Mass production of precast concrete floor joists (in Rumanian)

Industria Constructiilor si a Materialelor de Constructii (Bucharest), V. 6, No. 1, Jan. 1955, pp. 62-63
Reviewed by J. J. Polivka

Special "balancing" wood forms were used for mass production of floor members having shape of inverted U, 14-16 in. wide. Vibration was produced by rocking the forms on semi-circular supports, and each form was good for approximately 350 joists. Of special interest is technique for manufacturing T-joists during winter. The forms were immersed in electrically-heated salt solution.

Prestressed Concrete

Economic advantages of prestressed concrete (in Dutch)

B. Visser, *Cement* (Amsterdam), V. 7, No. 9-10, Oct. 1955, pp. 219-221
Reviewed by John W. T. Van Erp

Summarizes papers presented on this subject at the 2nd Congress of the F.I.P. (International Federation of Prestressed Concrete) at Amsterdam. Papers unanimously emphasize saving in cost compared to conventional concrete ranging from 5 to about 32 percent. Biggest savings were in dead load, important in long-span structures. Other advantages emphasized, most of them due to the absence of hair cracks were: great durability, imperviousness to chemical attack, resistance to sea water, minimum maintenance, and suitability for prefabrication due to light weight.

Properties of Concrete

Computation of freezing resistance of concrete at early ages

P. Nerenst, *Reprint* No. 73, Danish National Institute of Building Research, 1956, 40 pp.
Author's Summary

The theory of hydraulic pressure due to freezing is applied to air-entrained concrete after a degree of hydration has been obtained, which prevents the formation of ice lenses in macroscopic scale. Various factors influencing the magnitude of the hydraulic pressure are discussed. Formulas are developed to compute the change of physical composition of cement paste during the hydration process, which in turn causes changes of the physical properties of the cement paste. All formulas are based upon the amount of non-evaporable water or derived functions and initial composition of the concrete. The formulas are applied in one example. From an assumed air content the bubble spacing factor is determined. By successive computation of the hydraulic pressure—as function of the non-evaporable water content—the degree of hydration is determined above which the concrete will not be damaged by repeated freezing and thawing. Non-evaporable water content can not readily be measured, but it is assumed to be related to heat of hydration. The development of heat of hydration can be computed as function of time and temperature from isothermal tests with the particular cement used.

Treatment of concrete with silicon fluoride gas (in Dutch)

A. A. V. D. Vlist, *Cement* (Amsterdam), V. 7, No. 9-10, Oct. 1955, pp. 241-244
Reviewed by John W. T. Van Erp

The article goes extensively into the chemistry of the hardening of cement, its durability and deterioration under various chemical influences. Different protective methods are described and finally the treatment with SiF_4 gas. This method (patented under the name Ocrating in the Netherlands) protects the concrete by formation of substances with silicon-acid characteristics. Considerable resistance against aggression is built up, and its strength increased. Results of various test programs are mentioned indicating the value of the method.

Steam curing of concrete (in Dutch)

O. Graf, *Cement* (Amsterdam), V. 7, No. 9-10, Oct. 1955, pp. 249-250
Reviewed by John W. T. Van Erp

Two methods of steam curing, low and high pressure, both use saturated steam. Results of tests are given and the type of cement is shown to have great influence, as well as its age. Low-pressure steam curing generally is more effective if the mix is not

too dense and a porous aggregate is used. With high-pressure steam curing (8 hr in the kiln at about 115 psi and about 16 hr in air) it was found that flexural strength was four times as high and compressive strength five times as high as for moist air curing. The shrinkage on changing from a saturated to a dry (50 percent relative humidity) state was found to be a maximum of 0.0005. Advantages of steam curing are elaborated upon, but it remains imperative that properties of cement, sand, and aggregates be investigated and their suitability for steam curing established.

Structural Research

Determination of the stresses in an arch dam from a rubber model

LETITIA CHITTY and ALFRED J. S. PIPPARD, *Proceedings*, Institution of Civil Engineers (London), Part I, V. 5, No. 3, May 1956, pp. 259-275
Reviewed by ARON L. MIRSKY

Describes method of obtaining values of vertical and hoop stresses in arch dams due to water pressures from measurements of radial and tangential displacements on rubber models. Measurements must first be corrected for the difference in value of Poisson's ratio, using method outlined in earlier paper (see Current Reviews, Nov. 1956, p. 557). Since results in general show good agreement with those obtained by time-consuming relaxation analysis, experimental approach offers good design tool.

Paper also discusses two other, less general, methods for analyzing model displacements, and gives approximate analytical methods for obtaining stresses due to gravity and temperature, for which model analysis cannot be used.

General

Irrigation engineering. V. 2—Projects, conduits, and structures

IVAN E. HOUK, John Wiley & Sons, Inc., New York, 1956, 531 pp., $14

Presents information on irrigation practices; the design, construction, and operation of various structures ranging from the smallest to the largest Bureau of Reclamation structures. It is a collection of up-to-date information, probably available elsewhere only through scattered technical articles in various periodicals. The emphasis is on the design for irrigation purposes rather than the use of concrete or the structural design. However, complete reinforced concrete flume designs in use are included; designs and details of large pipes; canal and tunnel linings; and many special structures, such as inverted siphons, culverts, control structures, and others, in reinforced concrete are presented. There is also a chapter on design of dams, including several types of concrete dams and concrete spillways for other dams. Would be suitable for use as either a textbook for student engineers or reference book for practicing designers.

Influence of weather factors on heat energy level—A case of calculation

JØRN JESSING, *Reprint* No. 73, Danish National Institute of Building Research, 1956, 64 pp.
AUTHOR'S SUMMARY

Weather factors affect the heat loss and heat supply of a structure. All forms of heat transfer are dependent on temperatures and temperature differences, which also determine the direction.

It is shown how it is possible to make a numerical calculation of heat emission by conduction, convection, and heat radiation, and of heat supply by natural radiation. All the methods can be used generally and not only in the case shown. It is suggested how to determine the temperature course in a construction from known data about heat emission and heat supply.

Deformations in the shells of rotary cement kilns

S. M. BRISBANE, *Mining Engineering* (Australia), V. 8, No. 2, Feb. 1956, 3 pp.

Reports an investigation measuring deformations in rotary cement kilns due to temperature and movement. Two types of deformations are reported: vertical movement of the kiln cross section as a whole, and deformations of the cross section itself. The results indicate that stiffening rings keep the hot kiln circular in their vicinity, and that a more frequent spacing of stiffening rings as well as any reduction of the bowing would relieve stress in the lining and increase the life of the refractory lining.